Research

Fields of speciality: Low Dimensional Topology, Geometric Topology, Geometric/Combinatorial/Homological Group Theory

Projects and Contributions

1. Low Dimensional Topology

(a) **Links, Virtual Links and Whitehead's Asphericity Conjecture.** The conjecture, formulated by J.H.C. Whitehead in 1941, states that a subcomplex of an aspherical 2-complex is aspherical. Whitehead's original motivation for working on this problem was that a positive answer would imply the asphericity of knot complements. Knot complements have long since been shown to be aspherical using 3-manifold techniques. However, Whitehead's Conjecture remains unanswered despite considerable effort. An ongoing project with Stephan Rosebrock has been the search for a combinatorial proof for the asphericity of the Wirtinger complex associated with a long knot ("long" means the ends of the knot are not connected). Such a proof would carry over to Wirtinger complexes of virtual long knots and ribbon disc complements. It follows from work of Howie that the asphericity question for virtual long knots is an important special case of the Whitehead conjecture. Indeed, a subcomplex of a 2-complex that 3-deforms to a point is homotopic to the Wirtinger complex of a virtual long knot. Using techniques from geometric group theory, we were able to show asphericity for prime alternating virtual long knots [1] .

1. J. Harlander and S. Rosebrock, *Generalized knot complements and some aspherical ribbon disc complements*, Journal of Knot Theory and its Ramifications, Vol. 12, No. 7 (2003) 947-962.
2. J. Harlander, *A class of hyperbolic ribbon disc complements*, submitted 2008.
3. J. Harlander and S. Rosebrock, *On distinguishing virtual knot groups from knot groups*, to appear in the Journal of Knot Theory and its Ramifications, 2009.

(b) **Wall's Domination Question.** Given a CW-complex it is natural to ask whether it can be replaced by a simpler one having the same homotopy type. Questions of this kind were first considered by J.H.C. Whitehead, who posed in particular the question, when is a CW-complex homotopically equivalent to a finite one? C.T.C. Wall answered this question by giving an algebraic characterization of finiteness. He also showed that a finite complex that is a homotopy retract of a finite n-complex has the homotopy type of a finite n-complex in case $n \geq 3$. The case $n = 2$ remains open, and is referred to as the D(2)-problem. This problem is closely related to other questions in topology and group theory, such as the geometric realization problem, the Eilenberg-Ganea problem and the relation gap problem. These matters are surveyed in [5]. There has been renewed interest in the relation gap problem due to both the infinite relation gap found by Bestvina and Brady (1998), and to recent examples given by Bridson and Tweedale (2008) which expand on earlier examples of David Epstein (1971). Presentations of free products of groups are at the center of many proposed relation gap examples. A new survey on free product presentations is currently in preparation [7].

1. J. Harlander, *Solvable groups with cyclic relation module*, Journal of Pure and Appl. Algebra 90 (1993), 189-198;
2. J. Harlander, *On perfect subgroups of one-relator groups*, in Proceedings of the Workshop on Geometric and Combinatorial Methods in Group Theory, ICMS Edinburgh, 1993, ed. A. Duncan, N. Gilbert, J. Howie, Cambridge University Press 1994;
3. J. Harlander, *Closing the relation gap by direct product stabilization*, Journal of Algebra 182 (1996), 511-521;
4. J. Harlander, *Embeddings into efficient groups*, Proc. Edinburgh Math. Soc. (40) (1997), 317-324;
5. J. Harlander, *Some aspects of efficiency*, in Groups-Korea 1998 (ed. Y.G. Baik, D.L. Johnson, A.C. Kim), de Gruyter 2000;
6. *Problems in Low Dimensional Topology*, with C. Hog-Angeloni, W. Metzler, S Rosebrock, in Kluwer Encyclopedia of Mathematics Supp. II (ed. Michiel, Hazewinkel), Kluwer Academic Publisher, 2000.
7. J. Harlander, *Small presentations of free products*, in preparation, 2009.

(c) **Homotopy Classification of 2-Complexes.** The goal is to classify all 2-complexes (up to homotopy) with a given Euler-characteristic and a given fundamental group G. This program has been completely carried out by W. J. Browning in case that G is a finite group satisfying Eichler's condition. In contrast hardly anything is known for the case where G is infinite. To my knowledge the homotopy classification has been carried out only for finitely generated free groups. In joint work with J. Jensen we study the case where G is a 2-dimensional group. It is known that for such groups there is only one homotopy type on the minimal Euler-characteristic level. In 1980 M. Dunwoody showed that the trefoil group admits more than one homotopy type one level up. It is unknown whether more than one homotopy type can occur if one moves up to higher levels. In current work with my student Andrew Misseldine we have begun working on the homotopy classification problem for the Klein bottle group. We have constructed an exotic algebraic 2-complex for the Klein bottle group with Euler characteristic one.

1. J. Harlander, J. Jensen, *On the homotopy classification of complexes with aspherical fundamental group*, with J. Jensen, Topology and its Applications, Vol. 153, Issue 15 (2006), 3000-3006.
2. J. Harlander, J. Jensen, *Exotic relation modules and homotopy types for certain 1-relator groups*, with J. Jensen, Algebraic & Geometric Topology, Vol. 6 (2006), 2163-2173

2. Group Theory

(a) **The F_n-Conjecture for Metabelian Groups.** A metabelian group is an extension of abelian groups. In 1980 R. Bieri and R. Strebel characterized the finitely presented metabelian groups among the finitely generated ones in terms of a polyhedral set of directions in the unit sphere. This solved longstanding questions of P. Hall on metabelian quotients of solvable groups. Typically, the techniques employed in the study of finiteness properties of metabelian groups (such as finite presentability) are an interesting mix of geometric group theory and commutative algebra. This theory has evolved into a theory of geometric invariants for arbitrary groups. Nevertheless, some hard questions remain unsettled in the metabelian case. One of the most prominent of these is the F_n-conjecture, which relates finiteness properties of the group to the geometry of the sigma set (the polyhedral subset of spherical directions mentioned above). The original work of Bieri and Strebel settled the F_2-conjecture. Robert Bieri and I proved the F_3-conjecture using techniques from combinatorial homotopy theory. In a joint project with W.A. Bogley, we are working on proving the conjecture for groups with finite sigma complement.

1. R. Bieri and J. Harlander, *A remark on the polyhedrality theorem for the Σ-invariants of modules over abelian groups*, Math. Proc. Cambridge Phil. Soc. 131 (2001), no. 1, 39-43;

2. R. Bieri and J. Harlander *The FP_3-conjecture for metabelian groups*, J. London Math. Soc. (2) 64 (2001), 595-610;

3. *The Σ^2-conjecture for metabelian groups: the general case*, with D. Kochloukova, J. Algebra 273 (2004), no. 2, 435–454.

4. J. Harlander and D. Kochloukova, *On the Σ^3-conjecture for metabelian groups*, J. of the London Math. Soc. (3) 67 (2003), 609-625;

5. J. Harlander and D. Kochloukova, *Building Resolutions*, Journal of Algebra 269 (2003), 632-651.

(b) **Subgroups of Groups with Good Finiteness Properties.** A celebrated theorem of G. Higman states that a finitely generated group is the subgroup of a finitely presented group if and only if it is recursively presented. Is there an obstruction for embedding a finitely presented group into one with higher finiteness properties F_n? W.A. Bogley and myself were able to show that the answer is negative for metabelian groups and $n = 3$. Settling the above question for $n \geq 4$ entails proving the F_n-conjecture for a special class of metabelian groups, connecting this research with the project discussed in (a).

1. W. A. Bogley, J. Harlander *Improving tameness for metabelian groups*, New York J. Math. 10 (2004), 1-8.