Math 187 Assignment V

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Induction proofs are "informal" in the sense that you can use high school mathematical knowledge appropriately, but be sure to clearly indicate the structurte of the proof: clearly identify the basis step, the induction step, the induction hypothesis, and the places where the induction hypothesis is used.

You have until Monday, February 9 to turn this in, but be aware that I may give out another handout this week (don't assume that you are free to delay until the weekend!)

1. Prove by mathematical induction that $n(n+1)(2n+1)$ is divisible by 6 for any natural number n.

   You may get additional credit if you can present a convincing argument *not* using mathematical induction for this assertion: you may use the fact that any natural number n is of one of the forms $3m$, $3m-1$ or $3m-2$, depending on its remainder on division by 3.

   It is obvious from the formula for sums of squares given in the book that this must be true, but neither of the proofs above should appeal to this.

2. Prove by mathematical induction that

   $$\Sigma_{i=1}^{n} 2i - 1 = n^2$$

   .

   You can get additional credit by finding $f(i)$ such that

   $$\Sigma_{i=1}^{n} f(i) = i^3.$$

Hint: use the telescoping sum property. But you might need to do some additional fiddling to get things right.

3. Give a complete proof for exercise 4.6.4. Use summation notation to state the result (the book uses dots) and use properties of summation notation appropriately in your proof.

4. Examine the Fibonacci sequence

$$1, 1, 2, 3, 5, 8 \dots$$

Determine which of them are odd and which are even. When you have found a pattern, prove by mathematical induction that your pattern is correct. Hint: your statement might talk about more than one successive Fibonacci number, or you might need to use one of the modified forms of induction that we discuss.