Math 187 Graph Theory Problems

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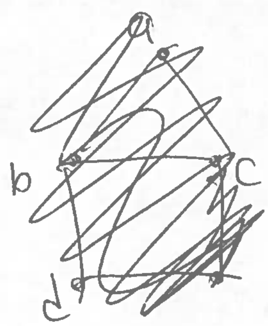
Later, I will post solutions to these.

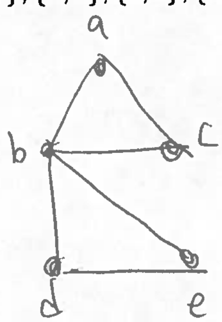
1. Definition of a graph

   The formal definition of a graph is "a pair of sets (V, E), where E is a set of two element subsets of V". But of course we really work with pictures with dots and connections between dots.

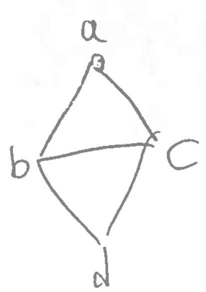
   Draw a picture of the graph whose official form is

   $$(\{a, b, c, d, e\}, \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{b, e\}, \{d.e\}\}).$$





   Write out the graph pictured below as a pair of sets.



$(\{a,b,c,d\}, \{\{a,b\}, \{a,c\}, \{b,d\}, \{c,d\}\})$

2. Write out the proof that the sum of the degrees of the vertices in a graph is even in your own words.

each edge meets exactly two vertices

so the ~~sotte~~ sum of all the degrees of the vertices counts each edge twice, that is, is twice the number of edges, and so is even.

3. For each of the following degree sequences, either draw a graph with just that number of vertices of just those degrees, or indicate why this isn't possible.

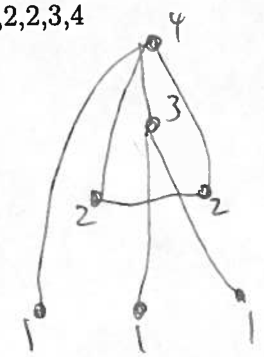
0,1,2,3,4

The degree 4 vertex must meet each other vertex, because there are only four others. But it can't meet the degree 0 vertex! (also, ~~each degree~~ some degree must occur twice)

1,1,2,2,3,4

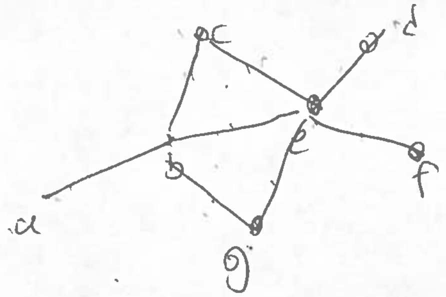
Impossible because there are an odd number of vertices of even degree

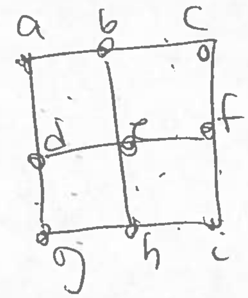
1,1,1,2,2,3,4



4. Computation of Euler's formula

Verify that Euler's formula $V - E + F = 2$ holds for the following connected planar graphs (connected means "all in one piece").

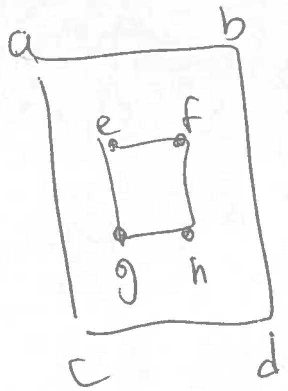




$V - E + F$
$7 - 8 + 3 = 2$

$V - E + F =$
$9 - 12 + 5 = 2$

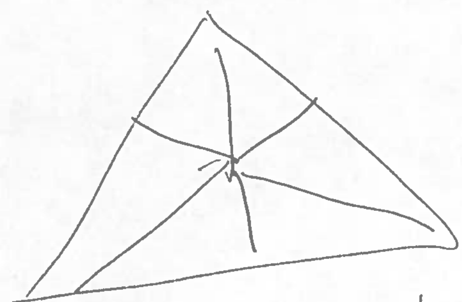
Verify that $V - E + F = 2$ does *not* hold for this graph. Why is this not a problem?



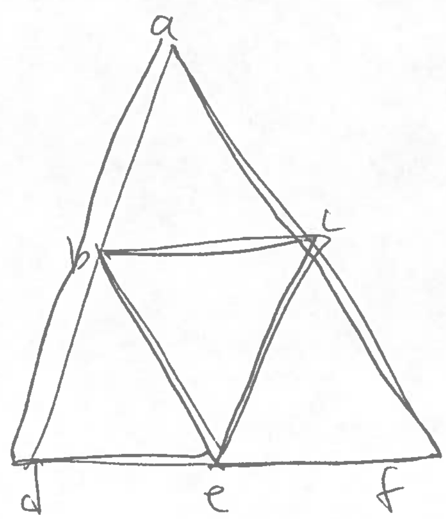
$V - E + F =$
$8 - 8 + 3 = 3$

but the graph is not "in one piece".

5. One of the following graphs admits an Eulerian walk (one can traverse all the edges in the graph in such a way as to visit each edge exactly once); one does not. Give a reason why you can't walk through the edges on one of them, and give an actual walk visiting all the edges for the other (represent the walk using the list of vertices you pass through, as I did in class).

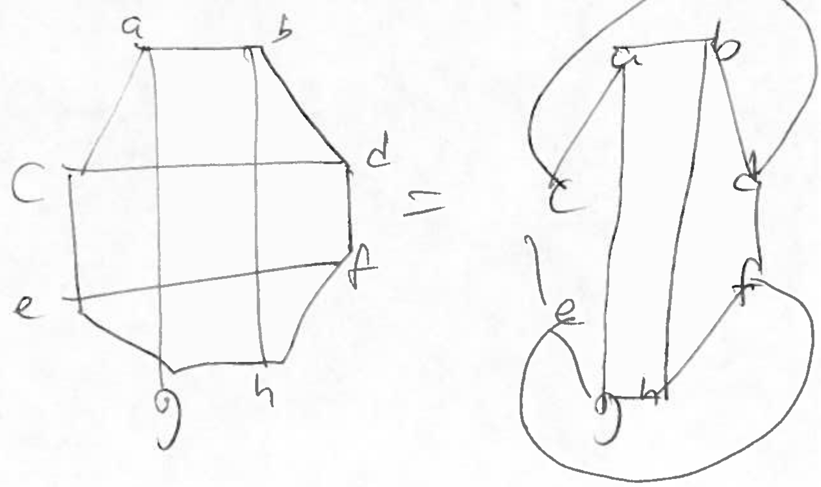


does not admit an Eulerian walk because there are six (>2) vertices of odd degree.

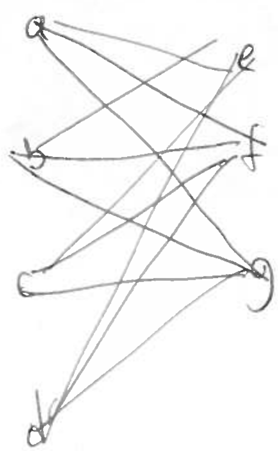


admits Eulerian walk, for example a, c, f, e, c, b, e, d, b, a

6. One of the following graphs is planar and one is not. For the one that is planar, draw a planar picture (the same vertices with the same connections but without any extra edge crossings). For the one that is not planar, give a calculation showing that it is not planar [the relevant formulas being $E \leq 3V - 6$ for any planar graph with at least three vertices, and $E \leq 2V - 4$ for a planar graph with at least three vertices with no triangles in it (meaning no three distinct vertices all connected to each other).



Same connections,
but planar



There are 7 vertices
and 12 edges.
There are no triangles
in this graph so if it were
planar we would have

$E \leq 2V-4$

$12 \leq 14-4 = 10$

which is not true -
so it is not planar.