# Math 187 Test IV

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The examination begins at 9:40 am and ends at 10:35 am. You may use a plain scientific calculator without graphing or symbolic computation capabilities. Cell phones must be turned off and out of sight.

You may expect that the question on which you do worst on this exam will be dropped.

1. Compute 38 `div` 10 and 38 `mod` 10. (`div` is integer division and `mod` is remainder.) That was easy!

   Compute -38 `div` 10 and -38 `mod` 10. That should be slightly less obvious.

2. Present the multiplication table for mod 5 arithmetic.

3. (a) Compute $\texttt{gcd}(11, 37)$. Express $\texttt{gcd}$(11,37) in the form $37x + 11y$.

(b) Compute the reciprocal of 11 in arithmetic mod 37. Be careful about signs. This uses the work of the previous part!

(c) Solve the equation

$$11x \equiv 7 \,\texttt{mod}\, 37.$$

4. Chinese Remainder Theorem

Find the smallest positive integer solution to the system of equations

$$x \equiv 5 \,\texttt{mod}\, 37$$

$$x \equiv 32 \,\texttt{mod}\, 42$$

What fact about 37 and 42 guarantees that there is a solution?

5. Permutation notation

   Let $\pi$ be the permutation $(13)(245)$ and $\sigma$ be the permutation $(2354)(1)$.

   Compute the composition $\pi \circ \sigma$ in table notation, then present it in cycle notation. Credit will be reduced if you compute $\sigma \circ \pi$ instead: be careful.

6. A specific group.

(a) Present the group table for $(\mathbb{Z}_{10}, \otimes)$ (multiplication in mod 10 arithmetic restricted to numbers relatively prime to 10).

(b) Identify the identity element of this group and the inverse of each element of the group.

(c) Find a generator for this group (show work indicating why it is a generator).

(d) Present an isomorphism between this group and the addition group in mod 4 arithmetic (this will be a 1-to-1 correspondence between elements of the two groups). I remind you that the elements of the addition group in mod 4 arithmetic are 0,1,2,3.

7. Complete the given partial group table. Write a brief explanation for why you placed each element (this can be quite informal): the format can be $a * b = c$ because... You are allowed to say things like "a given element can appear no more than once in a given column".

   Give a reason why this group is *not* isomorphic to mod 4 addition.

8. The operation table for the group $S_3$ is given below.

(a) Compute the order of each element of the group (the order of a group element is the smallest "power" of that group element which gives the identity).

(b) Present a subgroup of each of the following sizes, or give a brief explanation of why there cannot be one: 1,2,3,4.

(c) State a reason why this group is not isomorphic to the addition group of mod 6 arithmetic (there are several ways to see this; any one will do).