

# Fall 2015, Philosophy 209 Sample Test III

Dr. Holmes

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The test will begin at 10:30 and end officially at 11:20: I will actually give a five minute warning at that point. A summary of the rules is attached, including the quantifier rules.

You are allowed use of your test paper and your writing instrument. Cell phones should be turned off and out of sight.

General remarks about proofs: all propositional logic rules (including CP and RAA) may be used in any proof. You should not be certain that every premise is actually needed in every argument.

1. Proofs of theorems (arguments without premises)

Write a proof of the theorem

$$(P \rightarrow Q) \rightarrow \neg(P \bullet \neg Q)$$

Hint: since you have no premises, use CP and/or RAA to introduce some!

2. Supply line justifications in the proofs which follow.

The first example

**1**  $(\exists x)(Px)$  premise

**2**  $(\forall y.Py \rightarrow Qy)$  premise

**Goal:**  $(\exists z)(Qz)$

**3**  $Pa$

**4**  $Pa \rightarrow Qa$

**5**  $Qa$

**6**  $(\exists z)(Qz)$

The second example

**1**  $(x)(Px \vee Qx)$  premise

**2**  $(x)(Px \rightarrow Rx)$  premise

Goal:  $(x)(Qx \vee Rx)$

**3**  $Pa \vee Qa$

**4** Assume  $Qa$  for CP

**5**  $Qa \rightarrow Qa$

**6**  $Pa \rightarrow Ra$

**7**  $Qa \vee Ra$

**8**  $(x)(Qx \vee Rx)$

3. Identify invalid steps in proofs (give reasons when you say a line is invalid; you may just check a line which is valid without comment)

First example

**1**  $(\exists x)(Px)$

**2**  $(x)(Px \rightarrow Qx)$

**Goal:**  $(x)(Qx)$

**3**  $Pa$  EI 1

**4**  $Pa \rightarrow Qa$  UI 2

**5**  $Qa$  MP 3,4

**6**  $(x)(Qx)$  UG 5

Second example

**1**  $(x)(Px) \rightarrow (x)(Qx)$

**2**  $Pa$

**3**  $(x)(Rx \vee \neg Qx)$

**Goal:**  $Ra$

**4**  $Pa \rightarrow Qa$  UI 1

**5**  $Qa$  MP 2,4

**6**  $Ra \vee \neg Qa$  UI 3

**7**  $Ra$  DS 5,6

4. Proof with just UI and EG

Prove that the following argument is valid.

**1**  $(x)(Px \rightarrow Qx)$

**2**  $(x)(Rx \vee \neg Qx)$

**3**  $Pc$

**Goal:**  $(\exists w)(Rw)$

5. Proof with just UI and EG

Prove that the following argument is valid.

Hint: I think of using CP to prove some implications which will allow the conclusion to be drawn by constructive dilemma.

**1**  $(x)(Px \vee Qx)$

**2**  $Pa \vee Qb$

**3**  $(v)(Pv \rightarrow Rv)$

**Goal:**  $(\exists y)(Qy) \vee (\exists z)(Rz)$

6. Proof with EI

**1**  $(x)(Px \rightarrow Qx)$

**2**  $(\exists z)(\neg Qz)$

**3**  $(w)(Pw \vee Rw)$

**Goal:**  $(\exists x)(Rx)$

7. Proof with UG

**1**  $(x)(y)(Px \rightarrow Qy)$

**2**  $Pa$

**3**  $(z)(Rz \vee \neg Qz)$

**Goal:**  $(w)(Rw)$



8. Proof with non-quantified statements with quantifier components.

Show that the argument is valid.

**1**  $(x)(Px) \rightarrow (x)(Qx)$

**2**  $Pa$

**3**  $(x)(Px \rightarrow (w)(Rw))$

**4**  $(y)(Qy \vee \neg Ry)$

**Goal:**  $Ra$