

Name : _____

Quiz #3

Math 301, Spring 2013

Wednesday, May 8, 2013

1. Which of the following guarantee that the matrix $A \in \mathcal{R}^{n \times n}$ has an inverse? Circle all that apply.

- (a) A has a complete set of eigenvectors
- (b) The determinant of A is not zero,
- (c) A has at least one zero pivot,
- (d) There is exactly one solution to $Ax = b$.
- (e) The dimension of the nullspace of A is 0,
- (f) The columns of A span all of \mathcal{R}^n .
- (g) $\text{rank}(A) = n$.
- (h) A has at least one non-zero eigenvalue,
- (i) A is diagonalizable,
- (j) The reduced row echelon form of A is $R = I$.
- (k) A has n (non-zero) singular values.
- (l) A is similar to a diagonal matrix.
- (m) A has all positive eigenvalues.
- (n) A has real eigenvalues.
- (o) A is symmetric.
- (p) $\mathbf{x}^T A \mathbf{x} > 0$ for all non-zero $\mathbf{x} \in \mathcal{R}^n$.

2. How many eigenvectors do the following matrices have? Don't do any computations, but explain your reasoning.

$$(a) \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix}, \quad (b) \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad (c) \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$(d) \begin{bmatrix} 3 & 0 \\ 0 & -7 \end{bmatrix}, \quad (e) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (f) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$