Name : ______________________________

Homework #8

Math 301, Spring 2013

Due Wednesday, April 3, 2013

These homework problems are to be turned in and will be graded for credit. Turn in your work on separate pages, using this as a cover sheet. Please staple your work together. For full credit, you must show all of your work.

1. Suppose $A \in \mathcal{R}^{m \times n}$ and suppose that the columns of A sum to zero, and that the rows of A add to a row vector of 1's. Show that no such matrix can exist! **Note :** One answer to this problem is in the back of your book. Here you are asked to fill out the missing details and give an answer for the more general matrix A.

2. The **Fredholm Alternative** states that given $A \in \mathcal{R}^{m \times n}$ and $\mathbf{b} \in \mathcal{R}^{m \times 1}$, exactly one of the following is true :

   (a) The system $A\mathbf{x} = \mathbf{b}$ has a solution, or

   (b) $A^T\mathbf{y} = 0$ has a solution $\mathbf{y}$ with $\mathbf{y}^T\mathbf{b} \neq 0$.

   Given A below, find two vectors $\mathbf{b}$ which make each alternative true.

   $$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}, \qquad \mathbf{b}_1 = ?, \quad \mathbf{b}_2 = ?$$

   In which subspace is $\mathbf{y}$?

3. If $\mathbf{P}$ is the plane of vectors in $\mathbf{R}^5$ satisfying $x_1 + x_2 - x_3 - x_4 + x_5 = 0$, write a basis that spans $\mathbf{P}^{\perp}$. Construct a matrix that has $\mathbf{P}$ as its nullspace.

4. To find the projection matrix onto the plane $x - y - 2z = 0$, choose two vectors in that plane and make them the columns of A. The plane should be the column space. Then compute $P = A(A^TA)^{-1}A^T$.

5. In Problem 1, Section 4.3, you found the best fit line through the set of data points $b = 0, 8, 8, 20$ at $t = 0, 1, 3, 4$. Check that $\mathbf{e} = \mathbf{b} - \mathbf{p} = (-1, 3, -5, 3)$ is perpendicular to both columns of the matrix A you found in Problem 1. What is the shortest distance $\|\mathbf{e}\|$ from $\mathbf{b}$ to the column space A?