Name : ____________________________

Homework #7

Math 301, Spring 2013

Due Wednesday, March 20, 2013

These homework problems are to be turned in and will be graded for credit. Turn in your work on separate pages, using this as a cover sheet. Please staple your work together. For full credit, you must show all of your work.

1. Find a basis for each of these subspaces n $\mathcal{R}^4$.
   (a) All vectors whose components are equal.
   (b) All vectors whose components add to zero.
   (c) All vectors that are perpendicular to $(1, 1, 0, 0)$ and $(1, 0, 1, 1)$.
   (d) The column space and the nullspace of the 4×4 identity matrix $\mathbf{I}$.

2. Given a linearly independent set of vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, all in $\mathcal{R}^3$, construct a new set of vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ as follows :

$$\begin{aligned} \mathbf{v}_1 &= -2\mathbf{u}_1 + \mathbf{u}_2 \\ \mathbf{v}_2 &= \mathbf{u}_1 - 2\mathbf{u}_2 + \mathbf{u}_3 \\ \mathbf{v}_3 &= \mathbf{u}_2 - 2\mathbf{u}_3 \end{aligned}$$

   Show that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly independent.

3. If $\mathbf{V}$ is the subspace spanned by $(0, 1, 1)$ and $(2, 0, 1)$, find a matrix A that has $\mathbf{V}$ as its row space. Find a matrix B that has $\mathbf{V}$ as its nullspace.

4. If the entries of a 4×4 matrix are chosen randomly between 0 and 1, what are the most likely dimensions of the four subspaces? What if the matrix is 4×7?

5. Add the extra column $\mathbf{b}$ and reduce A to echelon form.

$$\begin{bmatrix} A & \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 2 & b_1 \\ 3 & 4 & b_2 \\ 4 & 6 & b_3 \end{bmatrix}$$

   From the $\mathbf{b}$ column after elimination, read of $m - r$ basis vectors in the left nullspace. Show that those y's are the combinations of rows that give zero rows.

6. Construct $A = \mathbf{u}\mathbf{v}^T + \mathbf{w}\mathbf{z}^T$ (where $\mathbf{u}$ and $\mathbf{v}$ are rank 1 matrices) are who column space has basis $(1, 2, 4)$, $(2, 2, 1)$ and whose row space has basis $(1, 0)$, $(1, 1)$. Write A as a 3×2 matrix times a 2×2 matrix.