Name : _____

Homework #6

Math 301, Spring 2013

Due Wednesday, March 13, 2013

These homework problems are to be turned in and will be graded for credit. Turn in your work on separate pages, using this as a cover sheet. Please staple your work together. For full credit, you must show all of your work.

1. True or false (with reason if true or example to show it is false).
   (a) A square matrix has no free variables
   (b) An invertible matrix has no free variables
   (c) An m by n matrix has no more than n pivot variables
   (d) An m by n matrix has no more than m pivot variables.

2. The plane $x - 3y - z = 12$ is parallel to the plane $x - 3y - z = 0$. One particular point on this plane is $(12, 0, 0)$. Write down an expression describing all points on the plane
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = ?$$

3. Construct a matrix whose column space contains $(1, 1, 5)$ and $(0, 3, 1)$ and whose null space contains $(1, 1, 2)$.

4. Suppose A and B are n by n matrices, and $AB = I$. Prove from $\text{rank}(AB) \leq \text{rank}(A)$ that the rank of A is n. So A is invertible and B must be its two-sided inverse. Therefore $BA = I$.

5. Write the complete solution as $\mathbf{x}_p$ plus any multiple of $\mathbf{s}$ in the null space:
$$\begin{aligned} x + 3y + 3z &= 1 \\ 2x + 6y + 9z &= 5 \\ -x - 3y + 3z &= 5 \end{aligned}$$

6. Explain why these are all false
   (a) The complete solution is any linear combination of $\mathbf{x}_p$ and $\mathbf{x}_n$.
   (b) A system $A\mathbf{x} = \mathbf{b}$ has at most one particular solution.
   (c) If A is invertible there is no solution $\mathbf{x}_n$ in the nullspace.