Name : ________________________________

Midterm Make-up

Math 301, Spring 2013

No calculators, books, notes, or neighbors are allowed on the exam. Please show all of your work to get full credit. Good luck!

1. (10 points) Use Gauss-Jordon elimination to find the inverse of the matrix

$$A = \begin{bmatrix} -2 & 1 & -1 \\ 6 & -2 & 2 \\ 0 & -2 & 1 \end{bmatrix}$$

2. (10 Points) Find all values of a and b for which the system

$$\begin{bmatrix} 3 & 5 & 1 \\ 0 & 2 & -1 \\ 0 & a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ b \end{bmatrix}$$

   (a) has exactly one solution
   (b) has no solutions,
   (c) has an infinite number of solutions.

3. (5 points) What is the angle between the vectors $\mathbf{u} = (1, 3, -2)$ and $\mathbf{v} = (-1, 0, 4)$? You do not have to given an exact answer, but provide the formula you would enter into your calculator if you wanted to get the exact solution.

$$\theta = ?$$

4. (5 points) Is there a value a which makes the two vectors $(1, -5, 2)$ and $(a, 3, a)$ perpendicular? If so, what is it?

5. (5 points) If the last corner entry of A is $A(2, 2) = 8$ and the last pivot of A is $U(2, 2) = 3$, what different entry $A(2, 2)$ would have made A singular? Hint : Think about the LU decomposition of A.

6. (5 points) Suppose that the entries of the 4×4 matrix A are given by $a_{ij} = 4(i-1)+j$. What is entry b_{23} of the matrix $B = A^T A$? Compute only this entry and nothing more!

7. (5 points) Let $\mathbf{u} = (1, -17, 2)$ and $\mathbf{v} = (3, -3, 7, 21)$ (both are column vectors). What is the row-reduced echelon form of the rank one matrix $\mathbf{u}\mathbf{v}^T$? How many pivot columns and how many free columns does the matrix have?

8. (5 points) Without multiplying matrices, find a basis for the row and column spaces of A :

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 \\ 1 & 2 & 2 \end{bmatrix}$$

What is the rank of A?

9. (15 points) Compute the LU decomposition of the matrix A, given by

$$A = \begin{bmatrix} 3 & -1 & 4 \\ -3 & 0 & -4 \\ 6 & 7 & -1 \end{bmatrix}$$

10. (15 points) Use the LU decomposition you found above, and forward and back substitution to solve the linear system

$$\begin{bmatrix} 3 & -1 & 4 \\ -3 & 0 & -4 \\ 6 & 7 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 14 \\ 2 \end{bmatrix}$$

11. (10 points) Given two unit vectors $\mathbf{u}$ and $\mathbf{v}$,

(a) Show that $(\mathbf{u}+\mathbf{v})$ and $(\mathbf{u}-\mathbf{v})$ are perpendicular.

(b) Suppose $\mathbf{v}\cdot(\mathbf{u}-\mathbf{v})=5$. What is $\|\mathbf{u}-\mathbf{v}\|$?

12. (10 points) Find a matrix P so that when A is multiplied on the left by P, the result is as follows.

$$A=\begin{bmatrix} -101 & 8 & 2 & 5 \\ 4 & -27 & 7 & 9 \\ 11 & 3 & -54 & 10 \\ 12 & 6 & -4 & 14 \end{bmatrix} \qquad PA=\begin{bmatrix} 11 & 3 & -54 & 10 \\ -101 & 8 & 2 & 5 \\ 12 & 6 & -4 & 14 \\ 4 & -27 & 7 & 9 \end{bmatrix}, \qquad P=?$$