Name : ______________________________

Midterm #1

Math 301, Spring 2013

Monday, Feb 25, 2013

No calculators, books, notes, or neighbors are allowed on the exam. Please show all of your work to get full credit. Good luck!

1. (10 points) What is the inverse of the following 2×2 matrix? Find it using any allowed method.

$$A = \begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$$

2. (10 points) Find a linear combination of three vectors $\mathbf{u} = (2, 2, 4)$, $\mathbf{v} = (1, 0, 0)$, and $\mathbf{w} = (0, 3, 0)$ that produces the vector $z = (7, 13, 8)$.

3. (10 Points) Find all values of a and b for which the system

$$\begin{bmatrix} 3 & 5 & 1 \\ 0 & 2 & -1 \\ 0 & -1 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ b \end{bmatrix}$$

(a) has exactly one solution

(b) has no solutions,

(c) has an infinite number of solutions.

4. (10 points) Find a matrix E such that EA is an upper triangular matrix. Then use this to find a lower triangular matrix L such that $A = LU$.

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & -3 & 4 \\ 0 & 6 & 1 \end{bmatrix}$$

5. (15 points) Compute the LU decomposition of the matrix A, given by

$$A = \begin{bmatrix} 3 & -1 & 4 \\ -3 & 0 & -4 \\ 6 & 7 & -1 \end{bmatrix}$$

6. (15 points) Use the LU decomposition you found above, and forward and back substitution to solve the linear system

$$\begin{bmatrix} 3 & -1 & 4 \\ -3 & 0 & -4 \\ 6 & 7 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 14 \\ 2 \end{bmatrix}$$

7. (10 points) Given two unit vectors $\mathbf{u}$ and $\mathbf{v}$,

   (a) Show that $(\mathbf{u} + \mathbf{v})$ and $(\mathbf{u} - \mathbf{v})$ are perpendicular.

   (b) Suppose $\mathbf{v} \cdot (\mathbf{u} + \mathbf{v}) = 2$. What is $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$?

   (c) What is the largest possible value of $\mathbf{v} \cdot (\mathbf{u} + 2\mathbf{v})$?

8. (10 points) Find a matrix P so that when A is multiplied on the left by P, the result is as follows.

$$A = \begin{bmatrix} -101 & 8 & 2 & 5 \\ 4 & -27 & 7 & 9 \\ 11 & 3 & -54 & 10 \\ 12 & 6 & -4 & 14 \end{bmatrix} \qquad PA = \begin{bmatrix} 11 & 3 & -54 & 10 \\ -101 & 8 & 2 & 5 \\ 12 & 6 & -4 & 14 \\ 4 & -27 & 7 & 9 \end{bmatrix}, \qquad P = ?$$

9. (10 points) Use Gauss-Jordon elimination to find the inverse of the matrix

$$A = \begin{bmatrix} -2 & 1 & -1 \\ 6 & -2 & 2 \\ 0 & -2 & 1 \end{bmatrix}$$

10. (10 points extra credit) Suppose you have the LU decomposition of a matrix A such that $LU = PA$, where P is a permutation matrix. Write down an expression for the inverse of A using the matrices L, U and P and their inverses. Show that what you found is actually the inverse of A by showing that $AA^{-1} = I$.