Name : ______________________________

Final

Math 301, Spring 2013

Wednesday May 15th, 2013

You are allowed one page of notes (front and back). Otherwise, no books, calculators or neighbors! Be sure to answer questions as completely as you can. Convince me that you know why you get the answers you do.

1. (10 points) Find the LU factorization of A :

$$A = \begin{bmatrix} 6 & 2 & 8 \\ 9 & 5 & 11 \\ 3 & 1 & 6 \end{bmatrix}$$

2. (10 points) Solve using $Ax = b$ using the LU factorization you found above. Note : You must use the factorization; don't just try to guess the solution or find it by some other means.

$$b = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}$$

3. (10 points) Find a complete solution to the linear system

$$x + 2y - 3z \quad = \quad 5$$

The solution to the system is a ___________ (point/line/plane) in $\mathcal{R}$_____ .

4. (10 points) Find a basis for the subspace spanned by the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 8 \\ 17 \end{bmatrix}, \quad \begin{bmatrix} 5 \\ 12 \\ 13 \end{bmatrix}$$

For full credit, you must show how you arrived at the solution, and why your solution is a basis.

5. (10 points) A matrix $A \in \mathcal{R}^{17 \times 4}$ has rank 4.

Fill in the following blanks with an integer value or a choice from an indicated set of values.

(a) The row space of A is in $\mathcal{R}$_____ and has dimension __________ .

(b) To get a complete basis for $\mathcal{R}$_____ , $C(A)$ can be extended with __________ additional vectors from __________ (one of the four fundamental subspaces).

(c) The matrix $A^T A$ is a matrix in $\mathcal{R}$_____ $\times$ _____ and is (singular/non-singular).

(d) The system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b}$ is in $\mathcal{R}$_____ , has at most (zero/one/infinitely many) solution(s).

(e) To project the vector $\mathbf{b} \in \mathcal{R}$_____ onto the column space of A, multiply $\mathbf{b}$ by the projection matrix

$$\mathbb{P} = \_\_\_\_\_\_\_\_\_\_ ,$$

where $\mathbb{P}$ is in $\mathcal{R}$_____ $\times$ _____ .

6. Big Bird and Cookie Monster are discussing the solutions to two linear systems

$$2x + y = 1$$

and

$$x - 2y = 0$$

Big Bird observes (correctly) that these two equations describe lines that are perpendicular to each other. To test this observation out, Cookie Monster considers the solution $\mathbf{u} = (1, -1)$ to the first equation and the solution $\mathbf{v} = (2, 1)$ to the second equation. He is dismayed to discover, however, that $\mathbf{u} \cdot \mathbf{v} \neq 0$. What should Cookie Monster have remembered from his Linear Algebra class that would help him understand this apparent contradiction?

7. (10 points) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

8. (10 points) The members of the crew from the TV series *Lost* have run out of wacky adventures on their island, and, having given up all hope of rescue, are now trying to remember what they learned in linear algebra. They start by trying to construct a diagonalizable matrix. Here is how far they have gotten :

$$A = \begin{bmatrix} -3 & 5 & 11 & -7 \\ b & -3 & a & \\ & 3 & 1 & \\ c & 6 & & 5 \end{bmatrix}.$$

Help them out by suggesting a possible strategy for guaranteeing that their final matrix is diagonalizable. Following your strategy, what values should they supply for a, b and c?

9. (10 points) Use the cofactor formula $A^{-1} = C^T / \det(A)$ to find the inverse of A.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

10. (10 points) Of the following conditions, only 6 guarantee that matrix $A \in \mathcal{R}^{n \times n}$ is invertible. Circle the 6 that apply.

(a) The determinant of A not zero.

(b) The row space of A has dimension $r < n$.

(c) $\text{rank}(A) = n$.

(d) A has a complete set of eigenvectors

(e) A has at least one non-zero eigenvalue,

(f) A has $r < n$ pivots,

(g) A is diagonalizable,

(h) The nullspace of A has dimension 0.

(i) A has real eigenvalues.

(j) The reduced row echelon form of A is $R = I$.

(k) A is symmetric.

(l) A has all positive eigenvalues.

(m) The linear system $Ax = b$ has exactly one solution.