Name :

Final Review - Math 170 - Fall 2011

1. Evaluate the following limits using any of the techniques we have learned in class.

   (a) $\lim_{z\to 9} \frac{\sqrt{z}}{z-7} =$

   (b) $\lim_{h\to 0} \frac{(1+h)^3 - 1}{h} =$

   (c) $\lim_{x\to 8} \frac{\sqrt{x-4}-2}{x-8}$

   (d) $\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{2}{1-x^2}\right) =$

   (e) $\lim_{\theta\to 0} \frac{\tan(\theta)}{\theta} =$

   (f) Using the definition of the derivative, show that the derivative of $f(x) = \sin(x)$ is $f'(x) = \cos(x)$. What trigonometric limits do you need?

   (g) $\lim_{x\to 2^+} f(x) = 5$ and $\lim_{x\to 2^-} f(x) = 2$. Does $\lim_{x\to 2} f(x)$ exist?

   (h) $\lim_{x\to\infty} \frac{9x^2 - 2}{6 - 29x} =$

   (i) $\lim_{t\to -\infty} \frac{4 + 6e^{2t}}{5 - 9e^{3t}} =$

   (j) $\lim_{t\to\infty} \frac{t^4}{e^t + 5} =$

2. Find the points on the graph of $y = x^2 + 3x - 7$ at which the slope of the tangent line is equal to 4.

3. Find the velocity of an object dropped from a height of 300 m at the moment it hits the ground.

4. A ball tossed in the air vertically from ground level returns to earth 4 s later. Find the initial velocity and maximum height of the ball.

5. Compute the derivatives of the following functions.

   (a) $y = 12e^{2x}$

   (b) $y = 4x^{5/3} - 3x^{-2} - 12$

   (c) $y = \sin(\cos(\sin(x)))$.

   (d) $y = \sin^{-1}(7x)$.

   (e) $y = (\tan^{-1}(x))^3$.

(f) $y = \ln(x^{-1})$

(g) $y = 7^{\sin(x)}$

(h) $y = (2x+1)^4(x-6)^3(x+7)^{11}$

(i) $y = \tanh(e^x)$

(j) $y = \sqrt{\cosh(3x)}$

(k) Find $\dfrac{dy}{dx}$ if $e^{xy} = \sin(y^2)$

6. Find the equation of the tangent line at the given point $xy + x^2y^2 = 6$ at $(2, 1)$.

7. The radius r and height h of a circular cone change at a rante of 2 cm/s. How fast is the volume of the cone increasing when $r = 10$ and $h = 20$?

8. A hot air balloon rising vertically is tracked by an observer located 4km from the lift-off point. At a certain moment, the angle between the observer's line of sight and the horizontal is $\pi/5$, and it is changing at a rate of 0.2 rad/min. How fast is the balloon rising at this moment?

9. Julien if jogging around a circular track of radius 50 m. In a coordinate system with the center of the track, Julian's x-coordinate is changing at a rate of -1.25 m/s when his coordinates are $(40, 30)$. Find dy/dt at this moment.

10. Find the linearization of the function $P(x) = e^{-x^2/2}$ at $a = 1$.

11. Find the critical points of the function $g(\theta) = \sin^2(\theta) - \cos(\theta)$ on $[0, 2\pi]$. Which are maximum, minimum or neither?

12. Find any minimum, maximum points, points of inflection, and regions where the function is concave up and concave down for the function $y = 2x^3 - x^2 - x$.

13. In Figure 1 are the graphs of a function $f(x)$, and its derivatives $f'(x)$ and $f''(x)$ (not in order). Label each graph to indicate which is $f(x)$, $f'(x)$ and $f''(x)$. Explain your reasoning. On the graph of $f(x)$, label any critical points, inflection points, and regions where the function is concave up and concave down.

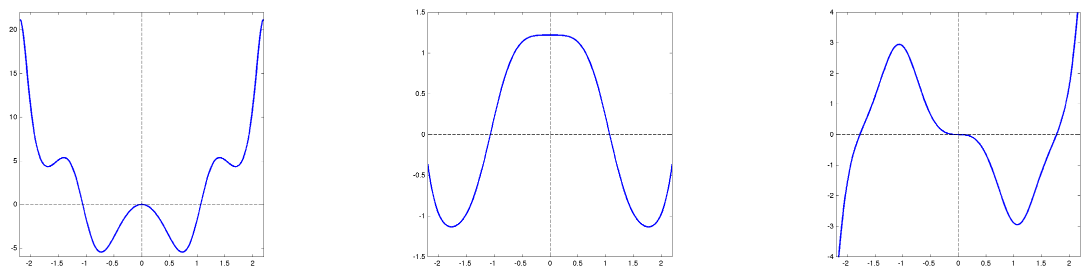

Figure 1: Graphs for Problem 13

14. Does the function $f(x) = x^4$ have an inflection point at $x = 0$? (Recall : At an inflection point, the sign of $f''(x)$ changes.)

15. Find a point P on the line $y = 4 - 3x$ closest to the point $(2, 0)$. Show that the line segment joining the closest point and $(2, 0)$ is perpendicular to the line.

16. A flexible tube of length 4m is bent into an L-shape. Where should the bend be made to minimize the distance between the two ends?

17. What is the anti-derivative of the function $f(x) = x\cos(x^2)$?

18. Evaluate $\displaystyle\int_{-1}^{1} \sin(\pi x)dx$

19. Evaluate $\displaystyle\int (2x-3)^5(2x+1)^2dx$

20. Find the two values of b for which the integral $\int_1^b (5x-2)dx = 0$. Show graphically what the two values mean.