the 8-bit result via expansion slot No. 2 of an Apple IIe computer.

```
1    A = 49312 : B = 49664
2    SA = 18000 : NDP = 5000
3    FOR J = 256 TO NDP + 255 : POKE A,O :
     POKE SA + J, PEEK(B) : NEXT
```

The POKE to 49312 produces a pulse for the start conversion pin. PEEK(B) is the resulting input datum, which is then stored at 18000 + J. The program acquires data at an impressive rate, in the neighborhood of 140 samples/s. Unfortunately, the loop does not run at a perfectly constant speed. Table I shows measured sampling rates for the various possible values of PEEK(B), the A-to-D output number.

Clearly, data streams consisting of large numbers of zeros and small values near zero may suffer from alarming cumulative errors in the timing of the data acquisition. Four courses of action can be suggested: (a) use machine language for the acquisition routine; (b) read time from a crystal clock, along with the data values; (c) adjust the electronics to use only the upper half of the A-to-D range; (d) reduce the sampling rate by padding the loop with steps that run at a speed independent of the data values.

One might ask why the loop for acquiring 5000 data points is made to run from 256 to 5255. It seems that the same loop with the simpler beginning, FOR J = 1 TO NDP, exhibits a 4% drop in frequency as J passes through the range 128–255. The simple expedient of starting the index at 256 circumvents this problem. It might be noted, however, that loops in general tend to vary their speed as their index progresses to larger values; be careful.

These idiosyncrasies of BASIC were discovered in the course of listening to the A-to-D converter with an amplifier and speaker, an old shop trick that still has its value in these days of logic analyzers. The precise sampling frequencies were measured with a Hewlett–Packard 5381A frequency meter counting for 10 s. Routines that perform the above functions were tried on a Commodore 8032 computer also, and the same effects were observed.

Table I. Sampling rate versus value of sample.

Value of PEEK(B)	Acquisition rate (samples/second)
1	135.1
2,3	136.4
4–7	137.7
8–15	139.3
16–31	140.8
32–63	142.3
64–127	143.8
128–255	145.2
0	146.7

On mathematical and physical ladders

M. Freeman and P. Palffy-Muhoray
Department of Physics, Capilano College, Vancouver, British Columbia, Canada

(Received 3 February 1984; accepted for publication 2 April 1984)

In elementary calculus textbooks,[1–4] a common example of a problem in related rates involves a sliding ladder. Typically, the base of the ladder of length L is moved along the floor at a fixed rate v_0 away from the wall against which the ladder is leaning. The problem is to find the rate v_y at which the top of the ladder slides down along the wall; the expected solution is $v_y = v_0 \cot\theta$ where θ is the angle that the ladder makes with the floor.

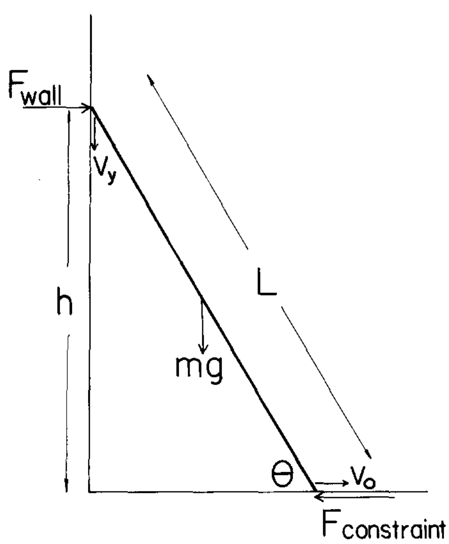

Fig. 1. Forces acting on the sliding ladder.

This solution is unphysical in that v_y tends to infinity as θ approaches zero, and it is therefore often puzzling to students. The purpose of this note is to point out that the top of the ladder loses contact with the wall when it is at a height $h = L(2v_0^2/3gL)^{1/3}$ above the floor, where g is the acceleration of gravity, and that v_y cannot be greater than $3^{1/4}(gL)^{1/2}$.

The physical ladder shown in Fig. 1 has mass m and it is acted on by gravity, normal contact forces at the wall and floor, and a constraint force required to maintain the uniform rate v_0. The net torque on the ladder about an axis through the base of the ladder and parallel to both wall and floor is

$$\tfrac{1}{2}mgL\cos\theta - F_{\rm wall}L\sin\theta = I\frac{d^2\theta}{dt^2}, \tag{1}$$

where $I = mL^2/3$ is the moment of inertia; this axis is fixed in an inertial frame moving with speed v_0 with respect to the wall. From the constraints $v_0 = d(L\cos\theta)/dt$ and $dv_0/dt = 0$, $d^2\theta/dt^2$ can be expressed in terms of v_0, L, and θ to

© 1985 American Association of Physics Teachers

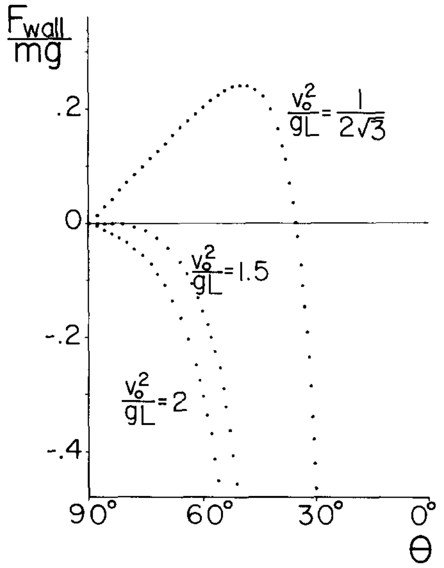

Fig. 2. Normalized contact force F_{wall}/mg versus angle θ for different values of the parameter v_0^2/gL. The maximum speed of the top of the ladder along the wall occurs when $v_0^2/gL = (2\sqrt{3})^{-1}$.

give

$$F_{wall} = \tfrac{1}{2} mg \cot\theta \, [1 - 2v_0^2/(3gL \sin^3\theta)]. \quad (2)$$

F_{wall}/mg is shown in Fig. 2 as a function of θ for different values of v_0^2/gL. If the ladder is leaning against the wall, F_{wall} must be positive. Depending on whether or not v_0^2/gL is less than 3/2, there are two cases to be distinguished.

If $v_0^2/gL \geqslant 3/2$, the top of the ladder loses contact with the wall as soon as the base is moved with speed v_0, regardless of θ. If $v_0^2/gL < 3/2$, it will remain in contact with the wall until the critical angle $\theta_c = \sin^{-1}[(2v_0^2/3gL)^{1/3}]$ is reached, corresponding to a height $h = L(2v_0^2/3gL)^{1/3}$ above the floor. At this time, the acceleration of the top of the ladder is $3g/2$ downward, while its speed is

$$v_y = v_0 \cot\theta_c = v_0[(3gL/2v_0^2)^{2/3} - 1]^{1/2}. \quad (3)$$

Finally, the maximum speed of the top of the ladder while it is still sliding down the wall, obtained from Eq. 3 by setting $dv_y/dv_0 = 0$, occurs when $v_0^2/gL = (2\sqrt{3})^{-1}$ or $\theta_c = 35.3°$, and equals $v_{y\,max} = 3^{1/4}(gL)^{1/2}$.

[1]R. C. Fisher and A. D. Ziebur, *Calculus and Analytic Geometry* (Prentice-Hall, Englewood Cliffs, NJ, 1975), p. 141.
[2]M. H. Protter and C. B. Morrey, *College Calculus with Analytic Geometry* (Addison-Wesley, Reading, MA, 1977), p. 183.
[3]E. J. Purcell, *Calculus with Analytic Geometry* (Appleton-Century-Crofts, New York, 1965), p. 160.
[4]S. K. Stein, *Calculus and Analytic Geometry* (McGraw-Hill, New York, 1973), p. 224.

Erratum: "Moment potentials" [Am. J. Phys. 52, 851–857 (1984)]

James G. Simmonds
Department of Applied Mathematics and Computer Science, University of Virginia, Charlottesville, Virginia 22901

Professor Harold Morton has kindly pointed out to me that the term

$$\tfrac{1}{4} P(\mathbf{M}\cdot\boldsymbol{\beta})\boldsymbol{\beta}$$

should be added to the left side of Eq. (14) of my recent article entitled, "Moment potentials" that appeared in Vol. 52 of the *American Journal of Physics* on pp. 851–857.

© 1985 American Association of Physics Teachers