Homework #7 (Supplement)

Math 426, Spring 2017

1. (a) Verify that $(z^\alpha)^n = z^{\alpha n}$ for $z \neq 0$ and integer n.
   (b) Discuss why in general $(z^{\alpha_1})^{\alpha_2} \neq z^{\alpha_1 \alpha_2}$ for $z \neq 0$.

2. A useful property of real numbers is $x^\alpha y^\alpha = (xy)^\alpha$.
   (a) Does the property $z^\alpha w^\alpha = (zw)^\alpha$ hold for complex powers?
   (b) Does the property $z^\alpha w^\alpha = (zw)^\alpha$ hold for the principal value of the complex powers?

3. Find all branch points for the following functions and choose branch cuts to make them single valued.

   This problem gives you an opportunity to explore more thoroughly the concept of a "branch". To determine branch points and branch cuts for the two given functions, consider the following :

   - Both functions are *composite* functions, i.e. $f(z) = h(g(z))$. So we have to consider branches of each function involved.
   - Find any points where the functions involved are undefined.
   - Find a domain over which the "outer-most" function is (a) single valued, (b) continuous, or (c) analytic. Use this to determine a branch over which the composite function is analytic.
   - Try out several values on your chosen branch and convince yourself that your function is single valued and continuous on the branch you have found.

   Good luck!

   (a) $f(z) = \ln(1 - z^{1/2})$
   (b) (**Math 526**) $f(z) = \ln(\ln(z))$