Homework #4 - **Hints!**

Math 426, Spring 2017

Due Friday 2/10/17

This homework assignment is your chance to re-do the problems from your exam. I would like everyone to re-do each problem. Use the hints below, or consult with each other. I will re-read the problems and give a score based on correctness *and* clarity. Even if you got full credit on a problem, most of you can improve your answer using the hints below.

1. Use the fact that $|z|$ is real to simplify some of the algebra. Also, note that $|z| = \sqrt{x^2 + y^2} = \sqrt{z\bar{z}}$ (several of you forgot the square root).

2. Draw a sketch of the three roots. You can leave your answer in terms of sines and cosines, or better, use the polar form $z = \rho e^{i\theta}$.

3. Assume a linear mapping of the form $f(z) = az+b$. Use the fact that z_0 is a fixed point of the mapping, i.e. $f(z_0) = z_0$. This gives you one of the unknown parameters in the mapping. What form does the second parameter have to take? Is there some arbitrariness in your choice? How does this affect the mapping?

4. Assume a linear mapping of the form $f(z) = az + b$. Show that a circle of radius r, centered at z_0 is mapped to a second circle. The problem is easiest if you use the polar form of the circle. Then, it is easy to come up with a formula that maps one circle to a second prescribed circle. Use this formula to obtain the mapping between the two specified circles. Is your mapping unique? Or did you have to make an arbitrary choice somewhere?

5. We did a very similar problem in class. Parameterize a general horizontal line as $z = x + ai$. Then, express $1/z = u(x) + v(x)i$. Work through the algebra to eliminate x to get an equation of a circle in (u, v) coordinates only. Your final answer should be a set in the complex plane.

6. Use the fact that $w = 1/z$ implies $z = 1/w$ and describe a set in the complex plane satisfied by w. Solve for w. What is the difference between images under the reciprocal function of circles that go through the origin and those that do not go through the origin?