

## Homework #3 (Supplement)

Math 426, Spring 2016

Turn in your written work in class. Undergraduates may do the **Grad student** problems for extra credit.

1. Illustrate the mappings  $f(z) = z^2$  by showing how this function maps horizontal and vertical lines in the domain of  $f(z)$  to the range of  $f(z)$ .
2. Illustrate the mapping  $f(z) = 1/z$  by showing how this function maps vertical and horizontal lines from the domain of  $f(z)$  to the range of  $f(z)$ .
3. **Grad students.** Illustrate  $f(z) = z + 1/z$  by showing how circles and lines are mapped under  $f(z)$ .
4. The plots shown in Figure 1 illustrate how circles and rays are mapped under  $f(z) = e^z$ .
  - (a) Describe as best you can, the range of angles (for rays) and radii (for circles) that were used to create the plot in the upper left of Figure 1.
  - (b) Let  $w_j = f(z_j)$ ,  $j = 1, 2, 3$  represent the three black solid dots that appear in plots 2-4. Determine the values for each  $w_j$  and their pre-image points  $z_j$ . Clearly show how you arrived at your conclusions.
  - (c) **Grad students.** Consider the parameterized curve  $\gamma(\theta) = (u(\theta), v(\theta))$ , shown by the thicker red line in plot 4. The distinguishing feature of this curve is that at the point  $w_4 = f(z_4)$ , shown by the solid black dot, the curve has 0 curvature. Determine parameterizations for the family of curves resulting from mapping circles under the mapping  $f(z)$ . Then, select the curve  $\gamma(\theta)$  with zero curvature at the point where it crosses the real axis. Determine the value of the point  $w_4$  and its pre-image point  $z_4$ . and obtain the parameterization for the curve  $\gamma(\theta)$ .

The following formula for curvature, given by

$$\kappa(\theta) = \frac{|\gamma'(\theta) \times \gamma''(\theta)|}{|\gamma'(\theta)|^3}$$

might be useful. Are there other ways to select the curve  $\gamma(\theta)$ ?

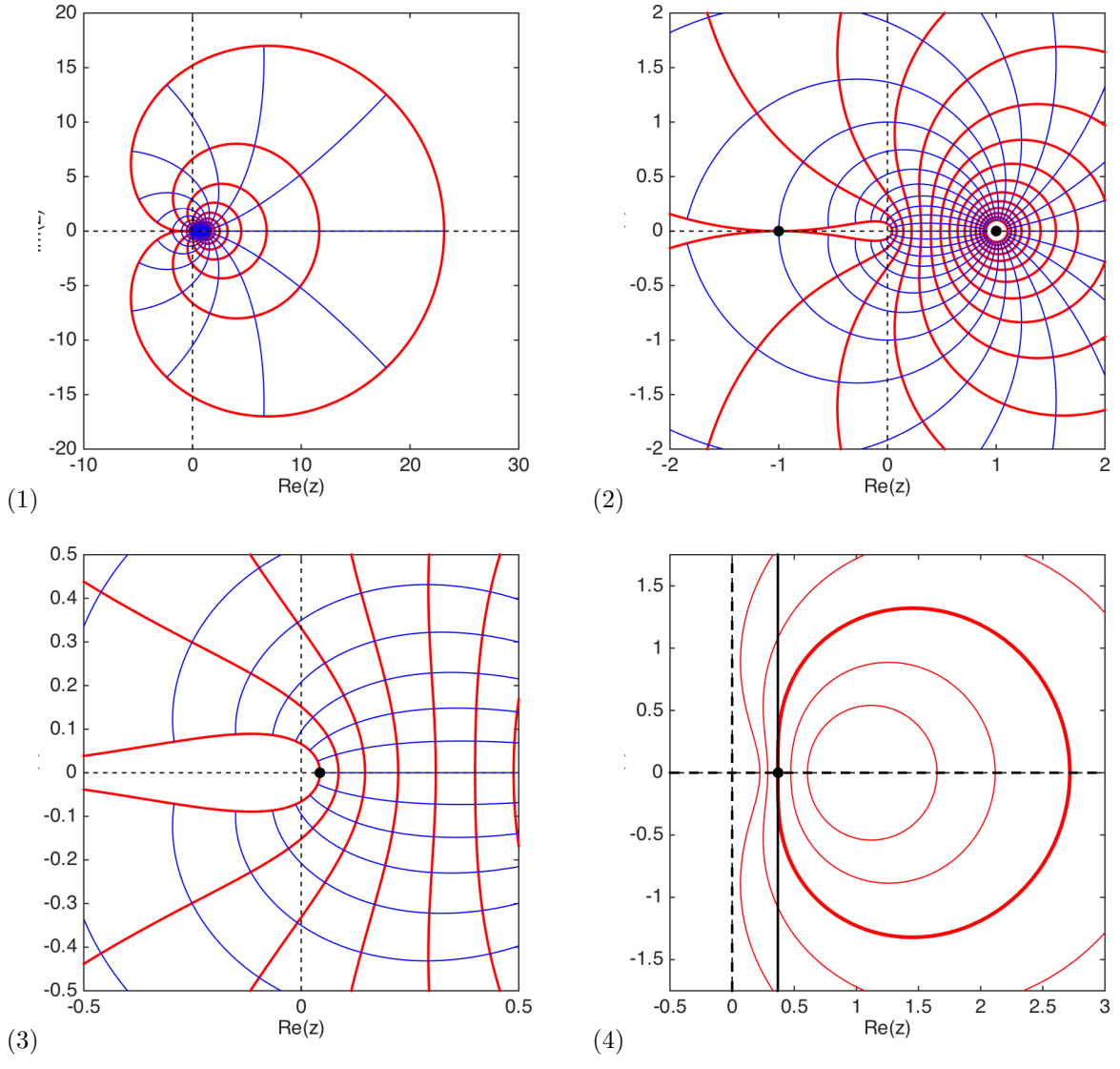


Figure 1: Images of circles and rays under the mapping  $f(z) = e^z$ .