# Linear algebra

Typical linear system of equations :

$$\begin{aligned} 5x_1 - x_2 + 2x_3 &= 7 \\ -2x_1 + 6x_2 + 9x_3 &= 0 \\ -7x_1 + 5x_2 - 3x_3 &= 5 \end{aligned}$$

The variables $x_1$, $x_2$, and $x_3$ only appear as linear terms (no powers or products).

This is a *square* linear system, since we have the same number of equations as variables.

# Linear algebra

Where do linear systems come from?

- Fitting curves to data
- Polynomial approximation to functions
- Computational fluid dynamics
- Network flow
- Computer graphics
- Difference equations
- Differential equations
- Dynamical systems theory

# Typical linear system

How does Matlab solve linear systems such as :

$$\begin{aligned} 5x_1 - x_2 + 2x_3 &= 7 \\ -2x_1 + 6x_2 + 9x_3 &= 0 \\ -7x_1 + 5x_2 - 3x_3 &= 5 \end{aligned}$$

- Does such a system always have a solution?
- Can such a system be solved efficiently for millions of equations?
- What does Matlab do if we have more equations than unknowns? More unknowns than equations?

# Solving linear systems

We are already familiar with at least one type of linear system

$$\begin{aligned} 3x_1 + 5x_2 &= 3 \\ 2x_1 - 4x_2 &= 1 \end{aligned}$$

The solution is the intersection of the two lines represented by each equation. This solution is a point $(x_1, x_2)$ that satisfies both equations *simultaneously*.

We could also view the solution as providing the correct linear combination of vectors $(3, 2)$ and $(5, -4)$ that give us $(3, 1)$.

$$x_1 \begin{bmatrix} 3 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ -4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

# Linear algebra - a 2x2 system

We can *row-reduce* an augmented matrix to find the solution :

*Use an elementary row operation to produce a "0" in the lower left corner.*

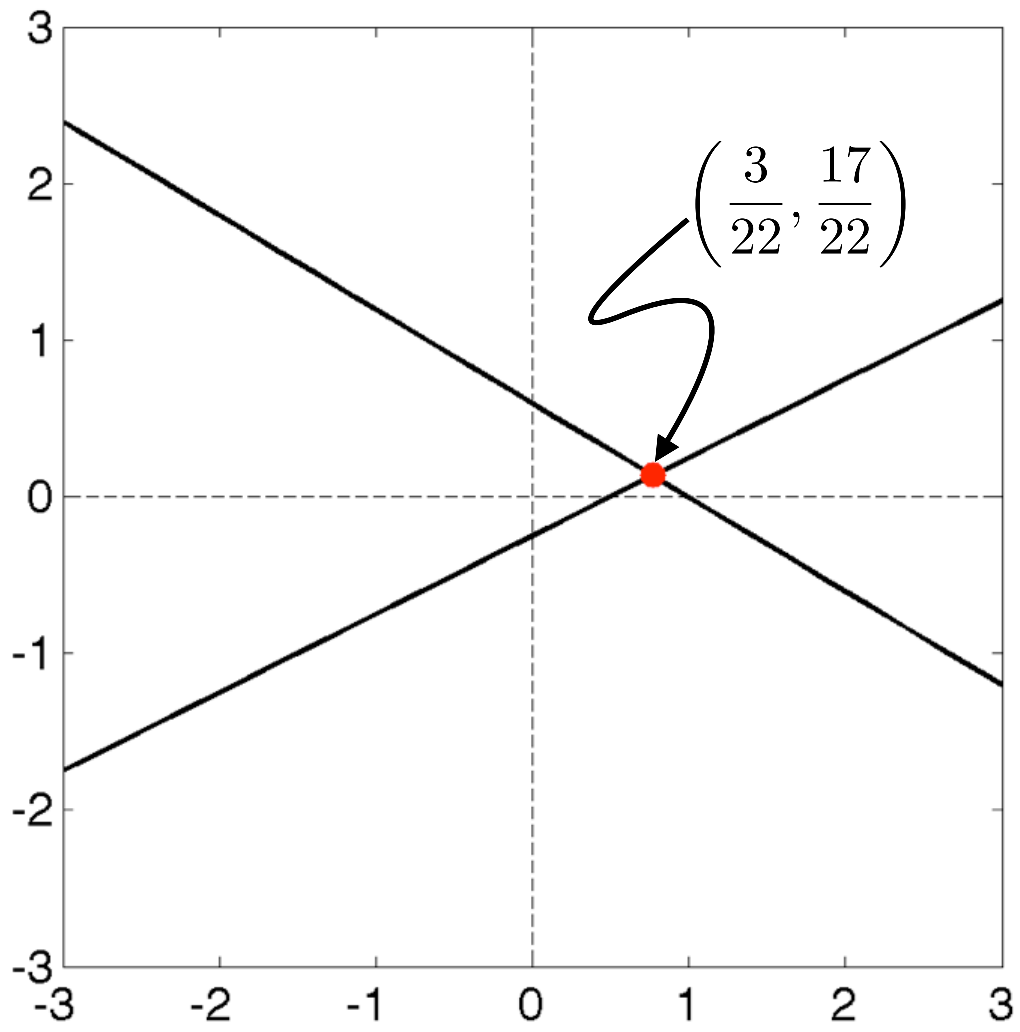
$$\left[\begin{array}{cc|c} 3 & 5 & 3 \\ 2 & -4 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 3 & 5 & 3 \\ 0 & -\frac{22}{3} & -1 \end{array}\right] \leftarrow \text{(eqn 2)} - \left(\frac{2}{3}\right) \text{(eqn 1)}$$

Use back-substitution to solve first for $x_2$ and then for $x_1$.

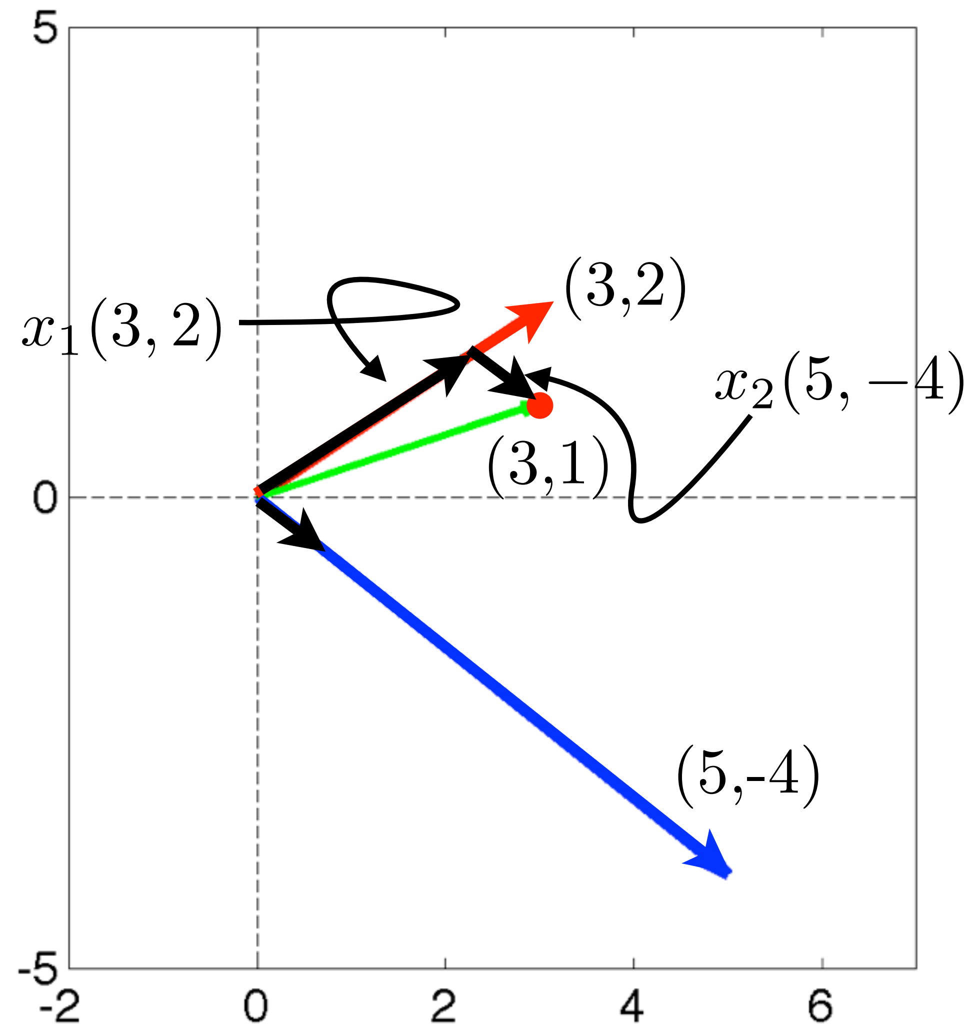
$$\begin{aligned} x_2 &= \left(\frac{-3}{22}\right)(-1) = \frac{3}{22} \\ x_1 &= \frac{1}{3}(3 - 5x_2) = \frac{1}{3}\left(3 - 5\left(\frac{-3}{22}\right)\right) = \frac{17}{22} \end{aligned}$$

Solution : $x_1 = \frac{17}{22}, \qquad x_2 = \frac{3}{22}$

# Linear algebra - a 2x2 system




Solution as the intersection of two lines




Solution as linear combination of vectors