Polynomial interpolation

Barycentric Formula for polynomial interpolation

Lagrange Formulation

The Lagrange form of the interpolating polynomial is given by

$$P_n(x) = \sum_{j=0}^{n} \ell_j(x) y_j$$

where

$$\ell_j(x) = \frac{\prod_{k=0,\ k\neq j}^{n} (x - x_k)}{\prod_{k=0,\ k\neq j}^{n} (x_j - x_k)}$$

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Lagrange interpolation

However, it is still expensive to compute Lagrange interpolating polynomial.

- Each evaluation of $P_n(x)$ requires $O(n^2)$ flops.
- Adding a new point (x_{n+2}, y_{n+2}) to be interpolated requires a whole new computation from scratch
- The method is numerically unstable.

Still a better method : Barycentric interpolation formula

Barycentric interpolation formula

Define $$\ell(x) = \prod_{k=0}^{n} (x - x_k)$$

This is not a Lagrange polynomial

Define $$\omega_j = \frac{1}{\prod_{k=0,k\neq j}^{n} (x_j - x_k)}$$

Denominator in the Lagrange Polynomial

Then $$\ell_j(x) = \ell(x) \frac{\omega_j}{x - x_j}$$

This is the jth Lagrange polynomial

and $$P_n(x) = \ell(x) \sum_{j=0}^{n} \frac{\omega_j}{x - x_j} y_j$$

First Barycentric Form

Barycentric interpolation formula

We have that

$$\sum_{j=0}^{n} \ell_j(x) = 1$$

We then have that

$$\ell(x) = \frac{1}{\sum_{j=0}^{n} \frac{\omega_j}{x - x_j}}$$

which leads to the form

$$P_n(x) = \frac{\sum_{j=0}^{n} \frac{\omega_j}{x - xj} y_j}{\sum_{j=0}^{n} \frac{\omega_j}{x - x_j}}$$

Second (true) Barycentric Form