

# Four Fundamental Subspaces

*The fundamental Theorem of Linear Algebra*

*Math 301*

# Four Fundamental Subspaces

- $C(A)$  is the column space
- $N(A)$  is the nullspace space
- $C(A^T)$  is the row space of  $A$ .
- $N(A^T)$  is the *left null space* of  $A$ .

New subspace!

The left nullspace is the space of all vectors  $\mathbf{y}$  that satisfy  $A^T \mathbf{y} = \mathbf{0}$ .

# Fundamental Theorem of Linear Algebra

Part 1

Let  $A \in \mathcal{R}^{m \times n}$  with rank  $r$ .

- The dimension of  $C(A)$  is  $r$
- The dimension of  $N(A)$  is  $n - r$
- The dimension of  $C(A^T)$  is  $r$
- The dimension of  $N(A^T)$  is  $m - r$

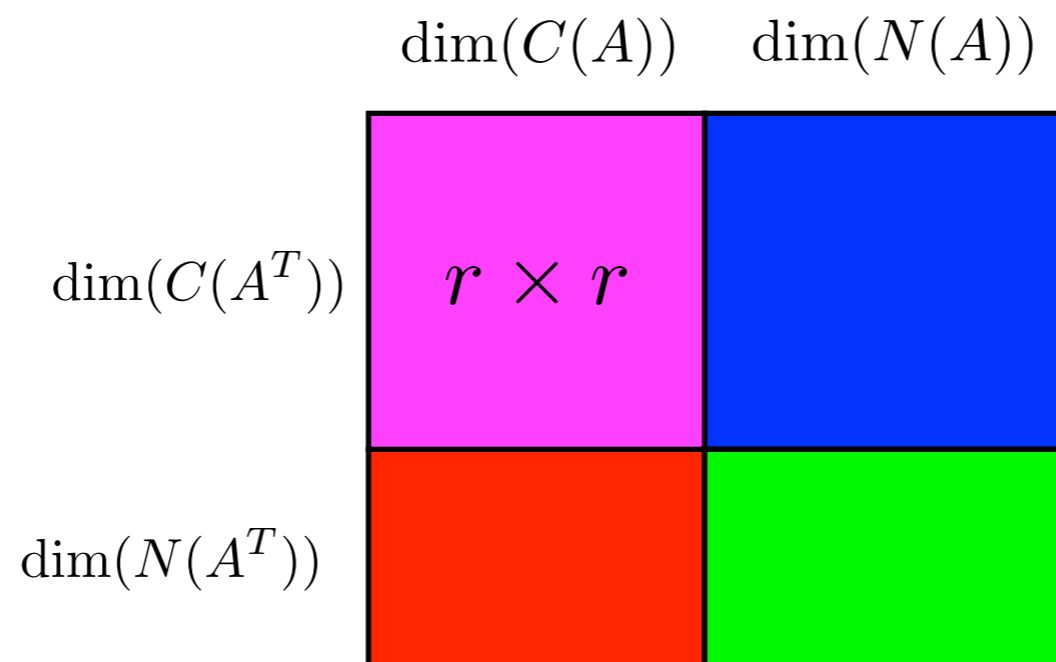
## Remember

$$C(A) \subset \mathcal{R}^m$$

$$N(A) \subset \mathcal{R}^n$$

$$C(A^T) \subset \mathcal{R}^n$$

$$N(A^T) \subset \mathcal{R}^m$$



# Four fundamental subspaces

## Question :

Suppose  $A \in \mathcal{R}^{7 \times 14}$  has rank 5. What are the dimensions of the four fundamental subspaces?

## Question :

The four fundamental subspaces for this matrix are in what four spaces?