

1. Find the determinants of each matrix in the sequence. Try to guess the rule that works for all matrices in the sequence. Use a calculator to confirm your guess in the 5×5 case.

$$(a) G_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, G_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, G_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \dots$$

$$(b) H_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, H_3 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, H_4 = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}, \dots$$

$$(c) P_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, P_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}, P_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}, \dots$$

(here each entry is the sum of the entry to the left and the entry above)

2. Problem set §4.3, exercises 1, 13, 15, 17, 24

3. Use Cramer's rule to find the inverse of each matrix (if it exists).

$$(a) \begin{bmatrix} 0 & 2 & -1 \\ 2 & 1 & 5 \\ -1 & 0 & 4 \end{bmatrix}$$

$$(b) \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & -2 & 1 & -1 \\ 0 & 2 & 2 & 3 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

4. Find the area of the parallelogram with sides $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$. Draw it and check your answer geometrically.

5. Find the volume of the tetrahedron through the origin and the three points $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

(Hint: a tetrahedron is half a parallelepiped.)

6. Problem set §4.4, exercises 18, 19, 20, 30, 31