

1. Determine whether each of the following sets of vectors are linearly independent or linearly dependent. In each case, remove those vectors that are not needed to describe the space spanned by the set.

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} \pi/3 \\ 2\pi/3 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ -3 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 0 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -4 \\ 6 \end{bmatrix} \right\}$

2. Find a basis for each of these subspaces in \mathbb{R}^4 .

(a) All vectors whose components are equal.

(b) All vectors whose components sum to zero.

(c) All vectors that are perpendicular to both $(1, 1, 0, 0)$ and $(1, 0, 1, 1)$.

(d) The column space of the 4×4 identity matrix \mathbf{I} .

(e) The null space of the 4×4 identity matrix \mathbf{I} .

3. Problem set §2.3, exercises 1, 2, 11, 22, and 29
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4. For each matrix, find bases and dimensions of all four fundamental subspaces.

(a) $A = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 2 & -6 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 2 & 6 & 1 & -4 \\ 1 & 3 & -2 & 3 \end{bmatrix}$

(d) $A = \begin{bmatrix} 2 & 0 & -1 & 4 & 2 \\ 4 & 1 & 0 & 2 & -1 \\ 2 & 1 & 1 & -2 & -3 \end{bmatrix}$

5. If \mathbf{V} is the subspace spanned by $(0, 1, 1)$ and $(2, 0, 1)$, find a matrix A that has \mathbf{V} as its row space. Find a matrix B that has \mathbf{V} as its nullspace.
6. If the entries of a 4×4 matrix are chosen randomly between 0 and 1, what are the most likely dimensions of the four subspaces? What if the matrix is 4×7 ?
7. Problem set §2.4, exercises 2, 3, 18, 20, and 31