

1. For each matrix  $A$  and vector  $\mathbf{b}$ , decide whether  $\mathbf{b}$  lies in the column space of  $A$ .

$$(a) A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1.5 \\ -3 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 1 & -3 & -3 \\ 2 & 1 & -4 \\ 3 & -1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -2 \\ -7 \\ 2 \end{bmatrix}$$

$$(c) A = \begin{bmatrix} -2 & 1 & 4 \\ 0 & -5 & -4 \\ 1 & -3 & -4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

2. Construct a  $3 \times 3$  matrix whose column space contains  $(1, 1, 0)$  and  $(1, 0, 1)$  but not  $(1, 1, 1)$ . Construct a  $3 \times 3$  matrix whose column space is only a line.

3. Problem set §2.1, exercises 2, 3, 22, 25, 27

4. For each of the following matrices, describe its null space as a linear combination of special solutions.

$$(a) A = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 4 & 1 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 1 & 2.5 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$(c) A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \\ -2 & -3 \end{bmatrix}$$

$$(d) A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \\ -2 & -6 \end{bmatrix}$$

5. For each system, use elimination to decide whether it is consistent. If it is, find the particular solution and all special solutions, and then write the general solution.

$$(a) \begin{cases} 2x + 3y - z = 1 \\ x + z = 1 \end{cases}$$

$$(b) \begin{cases} x + 3y + 3z = 1 \\ 2x + 6y + 9z = 5 \\ -x - 3y + 3z = 5 \end{cases}$$

6. The plane  $P$  defined by  $x - 3y - z = 12$  is parallel to the plane  $P'$  defined by  $x - 3y - z = 0$ . One particular point on the plane  $P$  is  $(12, 0, 0)$ . Describe all points on the plane  $P$  in parametric form.

7. Problem set §2.2, exercises 3, 4, 7, 8, 12, 30, 39.