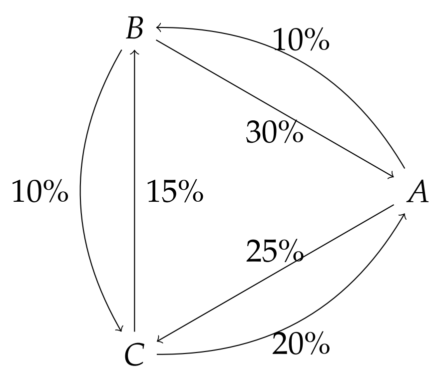
1. Gibonacci numbers (Exercise 4, Section 5.3). Consider the sequence $G_n$ which begins with $0, 1$ and then each successive entry is the average of the previous two. That is, $G_{n+1} = (G_n + G_{n-1})/2$.
   (a) Write down a $2 \times 2$ matrix $A$ that performs this process.
   (b) Diagonalize your matrix $A$
   (c) Use your diagonalization to find a formula for $G_n$.
   (d) Use your formula to compute the limit of the sequence $G_n$.
2. Suppose that each year, 25% of Lineartown moves to Algebraville, and 15% of Algebraville moves to Lineartown (and there are no other towns). What is the Markov matrix describing this behavior? What are its eigenvalues and eigenvectors? What is the eventual population distribution?
3. Now suppose that there are three towns, with yearly population movements described below:




   First write the Markov matrix for these movements. Second, show using calculations (or a proof) that $\lambda = 1$ is an eigenvalue of the Markov matrix. Third, find the eventual populations of the three towns. **Hint:** To show that $\lambda = 1$ is an eigenvalue, show that $\det(A - I) = 0$. You can do this because you know the columns of $A$ sum to 1.
4. Problem set §5.3, exercise 1, 8, 10, 15, 26, and 27.

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5. For each of the following matrices $A$, find the matrix $e^A$.
   (a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 6 & -2 \\ 2 & 1 \end{bmatrix}$
6. Use your calculations from the previous question to solve the linear system of differential equations.

(a) $u' = u + v, \quad v' = 2v, \quad u(0) = u_0, \quad v(0) = v_0$

(b) $u' = 6u - 2v, \quad v' = 2u + v, \quad u(0) = v(0) = 30$

7. Problem set §5.4, exercises 1, 2, 6, 8, 36, 37, 38.