Real Rational knots

Rama Mishra
IISER Pune
India

Abstract

A Real rational knot of degree $d$ is an embedding of $\mathbb{R}P^1 \to \mathbb{R}P^3$ defined by $[t, s] \mapsto [p_0(t, s), p_1(t, s), p_2(t, s), p_3(t, s)]$ where $p_i(t, s)$ are homogeneous polynomials of same degree $d$, that do not vanish simultaneously. It is easy to see that all knots in $\mathbb{R}P^3$ are isotopic to some real rational knot. Real rational knots can be categorized in two groups: the one that lie completely in $\mathbb{R}^3$ and the one that intersect a plane at infinity. we call the first one as affine knots and the other one as projective knots. Real rational affine knots are same as our classical knots. Real rational knots are projective closure of maps $\mathbb{R} \to \mathbb{R}^3$ given by $t \mapsto (r_0(t), r_1(t), r_2(t), r_3(t))$ where $r_i(t)$ are rational functions. This talk will present a technique to construct a real rational knot of reasonably low degree which is ambient isotopic to a given affine knot. We will generalize it to obtain real rational knots isotopic to any projective knot.