For this activity we will be referring to the unit circle. Using the unit circle below, explain how you can find the sine of any given angle, both by the ratio (opposite over hypotenuse) and by the ordered pairs. Explain how you can find the cosine of any given angle both by the ratio (adjacent over hypotenuse) and by the ordered pair.

Explain how the angle measures $\pi/6$ and $5\pi/6$ are similar. Explain how they are different.

Explain how $\pi/3$ and $5\pi/3$ are similar. Explain how they are different.

Explain how $\pi/4$ and $-\pi/4$ are similar. Explain how they are different.

Explain how the similarities that you found can be used to find $\sin \pi/6$ and $\sin 5\pi/6$ more easily.
Reference Angles
The reference angle for any angle $\theta$ in standard position is the positive acute angle formed by the terminal side of $\theta$ and the $x$-axis. In the diagram below $\theta$ is the angle we are looking for and $\beta$ is the reference angle.
Explain some reasons why we would use a reference angle? Do they make any calculations easier?

Using reference angles is a way to make the calculation of the values of trigonometric functions at various angles simpler.

Name the reference angle for each of the following angles. Explain how you found these.
1. $30^\circ$
2. $135^\circ$
3. $240^\circ$
4. $330^\circ$
5. $-210^\circ$
6. $-300^\circ$

After having found the reference angle, evaluate the following expressions. How did knowing the value of the reference angle make your calculations easier?

1. $\sin 30^\circ$
2. $\cos 135^\circ$
3. $\tan 240^\circ$
4. $\csc 330^\circ$
5. $\cot (-210^\circ)$

6. $\sec (-300^\circ)$
**Coterminal Angles**
Two angles are coterminal if they are drawn in the standard position and both have their terminal sides in the same location. For example, the angles measuring 280° and 640° are coterminal.

Draw two coterminal angles.

Why are they coterminal?

Why would we want to study coterminal angles? How do they make solving problems easier?

Write the measure of two angles that are coterminal (give the measure in degrees).

Write the measures of two angles that are coterminal (give the measure in radians).

Explain a method to find two or more angles that are coterminal.

Explain a method to find an angle measure that is between 0 and $2\pi$ radians and is coterminal with the angle $-15\pi$ radians.

Let’s let $k =$ any integer. Write an expression in terms of $k$ that you could use to find any coterminal angle to $\pi/4$.

What is the coterminal angle to -10.6 between 0 and $2\pi$. Using the unit circle above, can you estimate $\sin (-10.6)$? Within your group, come up with the best answer and explanation.

In order to be successful in this course, we need to view sine, cosine, tangent, cosecant, secant and cotangent as functions—that is for every x-value there is a distinct y value that corresponds—that is we are performing an operation on an angle—that is when given the angle measure we finding the ratio that corresponds. For example, if I asked you to find $\sin 30^\circ$ you would say exactly $\frac{1}{2}$. You would be performing an operation on the angle measure of 30 degrees. You would be creating a right triangle with your angle of 30 degrees in the center of the unit circle and find the ratio of the opposite/hypotenuse of this right triangle created by the 30 degree measure. 30 degrees or theta is your input value and the output value of your function is $\frac{1}{2}$ or the ratio. Theta as input, ratio as output.
This semester we have calculated the 6 trig functions for various angle measures. When we are given one of the angle measures on our unit circle, we come up with an exact value that we usually have memorized. When our angle measure is not one of the common angle measures on our unit circle or a coterminal angle to one of the angle measures on our unit circle then we use our calculator to calculate the value. Or, we use our calculator to give us the ratio. For example, \( \cos 34^\circ \) gives us \___________. We have created a triangle with the degree measure 34 in the center of our right triangle and output is the ratio of adjacent/hypotenuse.

Could you find \( \sin 340^\circ \) without a calculator? Or at least a good estimate? If you can, you have a better understanding of trig operations as functions.

Here are some questions for you to answer:

1. Estimate \( \sin 340^\circ \) without your calculator.
   a) Explain how to find this both using the unit circle and the sine graph. Show your answer with diagrams.

   b) Once you have done this, calculate the value of \( \sin 340^\circ \) using your calculator. Were you close? Explain why or why not?

2. Which is greater \( \cos 40^\circ \) or \( \cos 41^\circ \)? Do not use your calculator. Explain your answer. Then check your answer with your calculator. Were you close?
3. Which is greater $\sin 40^\circ$ or $\sin 129^\circ$. Do not use your calculator. Explain your answer. Check your answer on your calculator.

4. For what values of $x$ is $\sin x$ decreasing? Explain.

5. For what values of $x$ is $\sin x$ increasing? Explain.

For 4 and 5 above, make sure you explain your answer both from the unit circle and the graph.

6. When your calculator calculates $\sin 20^\circ$, how do you think it does this?

More Exercises

1. Find $\tan \theta$, $\cos \theta$, and $\csc \theta$, where $\theta$ is the angle shown in the figure. Give exact values, not decimal approximations.
2. Suppose that \( \theta \) is an angle in standard position whose terminal side intersects the unit circle at \( \left( -\frac{8}{17}, -\frac{15}{17} \right) \). Find \( \tan \theta \), \( \csc \theta \), and \( \cos \theta \).

3. Suppose that \( \left( x, \frac{5}{6} \right) \) is a point in quadrant I lying on the unit circle. Find \( x \).

4. Let \( \theta \) be an angle in quadrant III such that \( \sin \theta = -\frac{3}{5} \). Find the exact values of \( \sec \theta \) and \( \cot \theta \).

5. Find the exact value of \( \cos \frac{11\pi}{6} \).

6. Find the reference angle of \( \frac{7\pi}{9} \).

7. Find the terminal point on the unit circle determined by \( \frac{5\pi}{3} \). Use exact values, not decimal approximations.

8. Find the terminal point on the unit circle determined by \( \frac{\pi}{4} \).