Right Triangle Trigonometry

We use right triangles to study trigonometry. In right triangles, we have found many relationships between the measures of the angles and the lengths of the sides of the triangle as well as relationships between the lengths of the sides. For example, if you remember Pythagorean’s Theorem where the length of the leg squared plus the length of the other leg squared is equal to the length of the hypotenuse squared \( a^2 + b^2 = c^2 \). For the right triangle shown below, the 6 trigonometric relationships between angle measures and lengths of sides are shown. For example, the cosine \( \cos \) of whatever the measure of theta is equal to the ratio of the length of side \( a \) to the length of the hypotenuse (side \( c \)).

Using the above triangle, fill in values for \( a, b, c \) and \( \theta \) that will work to make a right triangle. As you look for values that work, think about how many different combinations of \( a, b, c \) and \( \theta \) are possible. Discuss how many different combinations there are and why. Explain.

Also discuss strategies to go about finding these values.

Have each person in your group come up with different values for \( a, b, c \) and \( \theta \). Your group should come up with at least 3 different scenarios. Explain how you arrived at your values. Check the values of the other students in your group.
In trig we often look at two common right triangles. These triangles are the 30-60-90 degree triangles and 45-45-90 degree triangles. These two triangles are pictured below.

In the triangles below, write the radian measure for each angle next to the degree measure. After you have done this, use the Pythagorean Theorem and the 6 trig ratios to give possible lengths for the sides of each triangle. Explain how you arrived at these values. Again, each group member could come up with different values. Are all your answers correct? Why or why not?

**Putting the triangle in the unit circle**

Using the triangles above, let RN and AB equal 1. Next, we are going to put the triangles into a unit circle. Sketch a circle of radius 1. Sketch the triangle ABC into the circle, let angle B be the central angle thus AB is the radius.

Sketch a circle of radius 1. Sketch the triangle RMN into the circle, let angle N be the central angle thus RN is the radius.

Given the hypotenuse is 1 unit and using the 6 trig ratios, find the lengths of the other sides of both triangles.

Now, use the grid paper at the end of this activity to draw a unit circle. Draw a circle where 1 unit is equal to 10 squares (thus the radius of the circle is 1 but is 10 units long). Draw the x and y axis and label the central angle of the circle.

Using the unit circle, draw right triangles with the following angle measures as the central angles in the first quadrant: \( \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6} \). (Note that you have created 3 triangles with the same...
angle measures as ABC, RNM and RMN.) Discuss the similarities and differences between the 3 triangles. Are any of the triangles the same? Explain. Make sure that you talk about what “the same” means.

Label the lengths of the sides of each triangle. (Hint: use the triangles ABC and RMN from above)
Recall the 6 trig relationships at the beginning of the activity. Use them to evaluate the following functions: (Make sure you are finding exact values for these functions. You can check your answers with your calculator. If you do, make sure your calculator is in radian mode. Throughout this semester when using your calculator it will be important to make sure your calculator is in degree mode when necessary and radian mode when necessary.) It will be important to have these values memorized.

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\begin{align*}
\sin \frac{\pi}{4} & \quad \cos \frac{\pi}{4} & \quad \tan \frac{\pi}{4} \\
\sin \frac{\pi}{3} & \quad \cos \frac{\pi}{3} & \quad \tan \frac{\pi}{3} \\
\sin \frac{\pi}{6} & \quad \cos \frac{\pi}{6} & \quad \tan \frac{\pi}{6}
\end{align*}
\]

**The Unit Circle**

Below is a picture of the unit circle. In the above portion of the activity, you should have drawn the above triangles into a unit circle like the one below. Notice that the ordered pairs on the coordinate grid correspond to the lengths of the sides of the triangles above. Discuss why this is.

You drew the triangles into the first quadrant. Try to draw the triangles into quadrants II, III and IV.
After you have drawn the triangles into the unit circle, label the quadrants I, II, III, and IV.

Looking at the ordered pairs around the circle, do you see any patterns? List them.

Looking at the values of sin \( \theta \) on the unit circle, do you see any relation between the sine of each angle and the ordered pairs? Explain why this happens.

Looking at the values of cos \( \theta \) on the unit circle, do you see any relation between the cosine of each angle and the ordered pairs? Explain why this happens.

Finally, look at the values of the tangent of each of the values above and list any patterns that you see.

What if this was not a unit circle, would your value for sin \( \pi/6 \) still be 1/2? Explain.
Finally, list the sign (positive or negative) of all 6 functions in each quadrant. For example, in quadrant II, tangent is negative because tangent = opposite/adjacent and in quadrant II, the opposite side is positive while the adjacent side is negative.