Math 144
Activity #7
Trigonometric Identities

What is a trigonometric identity?

Trigonometric identities are equalities that involve trigonometric functions that are true for every single value of the occurring variables. They are equations that hold true regardless of the values of the angles being chosen.

In trigonometry we use many identities to help us evaluate trig expressions, simplify expressions, verify proofs and prove other identities. Often we are given identities and just accept that they are in fact true. In this activity we are going to derive a few of the identities using pictures.

Recall the Pythagorean Theorem that shows the relationships between the sides of a right triangle. Using the unit circle below; sketch a right triangle in quadrant I. Make sure that the triangle you draw is in standard position and label that angle theta. Label the sides of your right triangle as a, b, and c where a and b are the legs and c is the hypotenuse.

Now, label the point on the unit circle where your right triangle intersects the unit circle. Label this point as (x, y). Now write both the x and y coordinates in terms of theta. This will give you an equivalent ordered pair for the marked point, but in terms of theta only.

Use the Pythagorean Theorem to write an expression in terms of theta.
This is one of the Pythagorean Identities.

Using Algebra, rewrite this identity in three different ways.

Now, take your original Pythagorean Identity and change the form of the identity so that you have the tangent function in the identity. Here is one more of your trig identities.

Finally, take the original Pythagorean Identity and change the form of the identity so that you have the cotangent function in the identity. Here is one more of your trig identities.

**Sum and Difference Formulas**

Using the unit circle below, draw a point in quadrant I and label it as B then draw a point in quadrant II and label it as A. Label the coordinate (1,0) on the unit circle below.

Let s be the length of the arc between points A and B.
Let u be the length of the arc from the point (1, 0) to point A.
Let v be the length of the arc from the point (1, 0) to point B.

\[ s = \text{__________} \text{ in terms of } u \text{ and } v. \]

What are the coordinates for the points A and B in the above circle? (Write them in terms of u and v.)
You should have found: A \((\cos u, \sin u)\) and B \((\cos v, \sin v)\)

Now rotate the circle so that point B has the coordinate of \((1, 0)\). You must keep the distance between A and B the same. Label your new A point as \(A'\) and your new B coordinate as \(B'\).

What are the coordinates for the points \(A'\) and \(B'\) in the above circle?

You should have found: \(A' \ (\cos s, \sin s)\) and \(B' \ (1, 0)\)

In both pictures, the chord (a line segment that contains the two points) between points A and B and the chord between points \(A'\) and \(B'\) are of equal length. Make sure you can see this. Explain why this is true.

Draw both of the chords in on your circle.

Since they are the same length and we have coordinates for each of the points, we can find the distance between them and they will be equal.

Find the distance between the points A and B:

\[
d(A, B) = \sqrt{(? - ?)^2 + (? - ?)^2}
\]

Now, simplify the right side of the equation as much as you can.

Next, find the distance between the points \(A'\) and \(B'\):

\[
d(A', B') = \sqrt{(? - ?)^2 + (? - ?)^2}
\]

Simplify the right side.

Since we know that \(d(A, B) = d(A', B')\) set the two expressions above equal to each other and solve for \(\cos s\). Recall from the previous page that \(s = u-v\).
Once you are done, you should get:

\[ \cos(u - v) = \cos u \cos v + \sin u \sin v \]

which is the identity we were looking for. The other sum and difference formulas can be found similarly.

**More Practice Using the Identities**

1. Find the exact value of \( \sin 15^\circ \).

2. Simplify \( \cos \frac{x}{2} \).

3. Given \( \sin x = \frac{1}{5} \) and \( \sin y = \frac{2}{3} \), angle \( x \) is in quadrant II and angle \( y \) is in quadrant III, find the exact value of \( \sin(x + y) \).

4. Find the exact value of \( \sin 50^\circ \cos 10^\circ + \sin 10^\circ \cos 50^\circ \).

5. Suppose \( \sin a = \frac{4}{5} \) and \( \sin b = \frac{5}{13} \), where both \( a \) and \( b \) are in quadrant I. Find \( \cos(a - b) \).

6. Verify that \( \cos(x - \frac{\pi}{2}) = -\cos x \) (make sure to explain what “verify” means).

7. Simplify \( \frac{\cos(x + y) + \cos(x - y)}{\sin x \cos y} \).
8. Suppose $\csc a = \frac{5}{3}$, where angle $a$ is in quadrant II. Find $\tan (2a)$.

9. Suppose $\cos b = \frac{5}{13}$, where angle $b$ is in quadrant IV. Find $\cos \left( \frac{b}{2} \right)$.
# MLC Approved Math 144 Identity Sheet

For students to use on Exam 2 and the Final Exam

<table>
<thead>
<tr>
<th>Identity</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}})</td>
<td>(\sin u \cos v = \frac{1}{2} \left[ \sin(u + v) + \sin(u - v) \right])</td>
</tr>
<tr>
<td>(\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}})</td>
<td>(\cos u \sin v = \frac{1}{2} \left[ \sin(u + v) - \sin(u - v) \right])</td>
</tr>
<tr>
<td>(\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u})</td>
<td>(\cos u \cos v = \frac{1}{2} \left[ \cos(u - v) + \cos(u + v) \right])</td>
</tr>
<tr>
<td>(\sin^2 x = \frac{1 - \cos 2x}{2})</td>
<td>(\sin u \sin v = \frac{1}{2} \left[ \cos(u - v) - \cos(u + v) \right])</td>
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<tr>
<td>(\cos^2 x = \frac{1 + \cos 2x}{2})</td>
<td>(s = r\theta)</td>
</tr>
<tr>
<td>(\tan^2 \theta = \frac{1 - \cos 2x}{1 + \cos 2x})</td>
<td>(s = \frac{1}{2} (a + b + c))</td>
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\[
A = \sqrt{s(s-a)(s-b)(s-c)}
\]

\[
s = \frac{1}{2}(a+b+c)
\]

\[
a^2 = b^2 + c^2 - 2bc\cos A
\]

\[
b^2 = a^2 + c^2 - 2ac\cos B
\]

\[
c^2 = a^2 + b^2 - 2ab\cos C
\]

\[
A = \frac{1}{2} ab\sin \theta
\]

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<td>(a^2 = b^2 + c^2 - 2bc\cos A)</td>
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<tr>
<td>(\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2})</td>
<td>(b^2 = a^2 + c^2 - 2ac\cos B)</td>
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<td>(\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2})</td>
<td>(c^2 = a^2 + b^2 - 2ab\cos C)</td>
</tr>
<tr>
<td>(\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2})</td>
<td>(A = \frac{1}{2} ab\sin \theta)</td>
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Draw the unit circle here