An exponential function is a function of the form \( f(x) = Ca^x \), where \( a \) is a positive real number and \( C \) is a real number. Also, \( a > 0 \) and \( a \neq 1 \), and \( C \neq 0 \). The domain of \( f \) is the set of all real numbers. The base \( a \) is the growth factor, and because \( f(0) = Ca^0 = C \), we call \( C \) the initial value. Exponential functions describe events that grow (or decline) at a constant rate.

The base of \( e \)
In the above description, we call \( a \) the base of our exponential. We also have said that \( a \) must be greater than 0, but not equal 1. What would happen if \( a \) was either 0 or 1? Verify your assumption.

One of the most common bases (\( a \) value) that we use is not a rational number—it is the real number \( e \). Locate \( e \) on your calculator. It is not a variable. Try entering \( e \) to the first power to see the approximation of \( e \).

There are several explanations of the history of \( e \). In 1683, Jacob Bernoulli looked at the problem of compound interest and, in examining continuous compound interest, he tried to find the limit of \( \left(1 + \frac{1}{n}\right)^n \) as \( n \) tends to infinity. He used the binomial theorem to show that the limit had to lie between 2 and 3 so we could consider this to be the first approximation found for \( e \). Also, if we accept this as a definition of \( e \), it is the first time that a number was defined by a limiting process.

In mathematics, the concept of a "limit" is used to describe the value that a function approaches (gets close to) as the input or index approaches some value. Limits are essential to calculus (and mathematical analysis in general) and are used to define continuity, derivatives, and integrals. The notation for a limit would look like this:

\[
\lim_{{n \to \infty}} \left(1 + \frac{1}{n}\right)^n
\]

And would be read the limit as \( n \) approaches infinity of \( \left(1 + \frac{1}{n}\right)^n \). To evaluate this we would make \( n \) as large as possible and see what the expression \( \left(1 + \frac{1}{n}\right)^n \) approaches or gets closer to. Try this with the expression \( \left(1 + \frac{1}{n}\right)^n \). As \( n \) gets larger what does the expression \( \left(1 + \frac{1}{n}\right)^n \) get closer and closer to?
The data set
Some roommates accidentally leave a banana without the peel on a counter. They go away for two days and come back to find a bunch of fruit flies swarming around the banana. (They did not observe that there were any fruit flies initially but this is not to say that there weren’t any.) They decide to count the fruit flies and come up with 123. Instead of cleaning up the banana, they are exhausted from counting fruit flies so they leave and forget about the banana. Not enjoying cooking very much, they do not enter the kitchen again for 2 more days. They then observe even more fruit flies. Again, they decide to count them and come up with 350. They now become interested in this fruit fly population so they purposefully leave them for another 3 days. When they return and count the fruit flies this time there are 1650 fruit flies. Finally they leave them another 2 days and the fly count is at 4590.
If they were to then leave for spring break and not return until the end of spring break (this would be 9 more days), how many fruit flies should they expect to be in their kitchen when they return?

The graph
First, plot the data set on the grid below. Make sure to label your x and y axis.

Looking at your plotted points, sketch a “curve of best fit.”

What type of equation do you think will model this graph best? What do you base your decision on? How can you prove or justify your conjecture?
Finding a Model
Go to the document labeled Regressions. Use the directions to graph the scatterplot on your calculator. Once you have a scatterplot on your calculator, check to make sure that your sketched scatterplot looks like the scatterplot on your calculator. Discuss why the graphs may or may not look the same.

Next, use the steps for finding a linear regression and find the linear regression equation that best fits the data. Does this fit the data well? Why or why not? What would happen to the population if the linear regression were the best fit?

Next, use the steps for finding the linear regression except choose ExpReg instead of LinReg. What is your exponential regression equation?

Graph this with the scatterplot. How well does this fit your data? Explain why you think it fits well or why it doesn’t fit well. (Write your equation in the form \( P(t) = ba^t \).)

Let’s look at one more regression equation before we move on. Looking at your scatterplot above, could you fit a cubic graph to the data? Explain why or why not. Include in your explanation the difference between an exponential and a cubic equation/graph.

What is the cubic regression that you found?

Graph the scatterplot, the exponential regression and the cubic regression on the same set of axis on your calculator and examine which one fits best. Support your decision.

Interestingly, when the data was gathered for this activity, it was gathered and modeled by an exponential equation with base \( e \). The mathematical equation that was originally found to model the fruit
fly population was \( P(t) = 44.6429e^{0.5148t} \). We treat \( e \) like a constant value and not as a variable. Compare and contrast your exponential equation with the equation that has base \( e \).

**Answering our question**
Assuming that they leave the fruit flies while they go on vacation, how many fruit flies will there be when they get home from their vacation? (Assume they will be gone for 9 days on their spring vacation.) Make sure you can explain how you arrived at your answer. Compare the answer you came to with your equation with an answer arrived at using my equation with base \( e \).

**Further exploring the data set**
Let’s use the equation \( P(t) = 44.6429e^{0.5148t} \) to answer the following questions:

1. What was the initial fruit fly population (Hint: Let \( t = 0 \))? How does this relate to \( C \) in \( f(x) = Ca^x \)?

2. After 3 weeks, how many fruit flies will there be?

3. Is there a limit to how large the fruit fly population will get? Explain your answer.

4. According to our model, 3 days before the roommates left the banana on the counter, how many fruit flies were in the house? Where is the data point on the graph located that indicates this point?

5. According to our model, 4 weeks before the roommate left the banana, how many fruit flies were on the counter?

**Scientific Notation on Your Calculator**
In question #5 above, your calculator gave you an answer that had an E in it. This E signifies scientific notation and denotes either really large numbers or really small numbers. In my case, my calculator gave 2.452776738 \( E \)-5. This means 2.452776738 \( \times 10^{-5} \) or 2.452776738 \( \times 0.00001 \) or 0.00002452776738—a very small number. So, 4 weeks earlier you would expect not even 1 fruit fly.
The Domain
Without using the situation, just looking at the equation, what is the domain of the equation? Explain why that is the domain.

What would be the domain in the fruit fly situation? Explain your reasoning.

The Range
Without using the situation, just looking at the equation, what is the range of the equation? Explain why that is the range.

What would be the range in the fruit fly situation? Explain your reasoning.

The Y-Intercept
Algebraically, we find the y-intercept of the graph by letting \( t = 0 \). Let’s use the base e function that represents the fruit fly population, \( P(t) = 44.6429e^{0.5148t} \).

Looking at the graph, what is the y-intercept?

Algebraically what is the y-intercept? Did you notice the connection between the constant \( C \) and the y-intercept?

The X-Intercept
Looking at your graph, what is the x-intercept? Explain your answer. You might want to make a connection between the range of the function and the x-intercept. Did you notice that the expression \( 44.6429e^{0.5148t} \) will never equal 0? Why is that?

Asymptote
After we looked at the x-intercept of the function, you might have noticed that you found the asymptote. What is the asymptote of the function \( P(t) = 44.6429e^{0.5148t} \)? How do you know this?
Algebraically, you find the asymptote when you examine what happens as \( t \) goes to negative infinity. Let \( t \) equal a large negative number such as -2000. Evaluate \( P(-2000) \). Next evaluate an even smaller number and record your result. Continue to do this and notice what happens to \( P(t) \) as you continue to use smaller and smaller values. Note, sometimes your calculator rounds answers off. For example, when I evaluate \( t = -2,000,000 \), my calculator gives me an answer of 0. You need to be careful when your calculator starts rounding answers. You will have to recognize that there is no number that you can put in for \( t \) that will make \( P(t) = 0 \).

**Connections**

Relating previous skills to a new skill: We plan to provide at least one problem on each test which expands previously learned skills to a new application. Today we will expand solving equations algebraically to solving them graphically.

Previously, you have learned how to solve the equation \( x^3 + x^2 + 2x - 3 = 0 \) algebraically by factoring and graphically by using the \( x \)-intercepts. You have also learned to factor \( 3x^2 \frac{3}{2} - 9x^2 + 6x \frac{1}{2} \). In this section, we will connect those with solving the equation \( 3x^2 \frac{3}{2} - 9x^2 + 6x \frac{1}{2} = 0 \).

First, solve the equation \( x^3 + x^2 + 2x - 3 = 0 \) both algebraically and graphically. Note the steps that you use to solve the equation algebraically. Also, note and list the steps that you used to solve the equation graphically making sure to note where your solutions lie on the graph.

Next, factor the expression \( 3x^2 \frac{3}{2} - 9x^2 + 6x \frac{1}{2} \).
Now, we will use those to solve the equation $3x^2 - 9x^2 = -6x^{-1}$ algebraically. In order to solve the equation algebraically, we solve it much like we solve $x^3 + x^2 + 2x - 3 = 0$. First we set it equal to zero and then we factor. It is important to set the equation equal to zero first. Then discuss what you would do next to solve $3x^2 - 9x^2 = -6x^{-1}$ . Discuss your solutions.

Finally, you can solve the equation $3x^2 - 9x^2 = -6x^{-1}$ in two ways. We have previously done this in two ways.