Previously, we have discussed supply and demand curves. At that time we used linear functions. Linear models are often used when introducing concepts in other subjects due to the simplicity of linear graphs. But are real situations best modeled with linear graphs?

Consider the supply curve. If we collect a few data points we might find a graph that looks like

Looking closely at that graph we find that initially there is a large increase in price as we increase the quantity. However, at the larger quantities, increasing the quantity has less of an effect on the price. If we draw a curve through the points, we obtain the following:

Looking at this graph, we should be able to recognize it as being similar to a square root graph. To make this easier to see, we will choose a scale that is easy to work with.
Consider the function $P_s = 2\sqrt{Q_s} - 1 + 3$. A few values of $Q_s$ have been given in the T-chart below. Compute the corresponding values for $P_s$.

\[
\begin{array}{c|c}
Q_s & P_s \\
1 & \\
2 & \\
5 & \\
10 & \\
\end{array}
\]

Consider the above points with the points on the graph above and it is easy to determine that they are the same values (if this is not clear, re-compute your values in the chart.).

Using a linear model for a demand curve we may have the function: $P_d = -\frac{1}{2}Q_d + 11$, and placing the supply and demand functions on the same graph, we have

\[
\begin{array}{c|c}
Q_s & P_s \\
1 & \\
2 & \\
5 & \\
10 & \\
\end{array}
\]

We now have a supply and demand equation and an equilibrium point somewhere between the quantity of 6 and 8.

Supply: $P_s = 2\sqrt{Q_s} - 1 + 3$
Demand: \( P_d = -\frac{1}{2} Q_d + 11 \)

Use the two equations to find the equilibrium point. The easiest process is to recall that at equilibrium we have \( P_s = P_d \). Using substitution method, we can now solve the equation:

\[
-\frac{1}{2} Q + 11 = 2\sqrt{Q - 1} + 3
\]

The first step is to subtract three on both sides of the equal sign:

\[
-\frac{1}{2} Q + 8 = 2\sqrt{Q - 1}
\]

Then we need to square both sides: (NOTE: we will refer to squaring both sides later)

\[
\left( -\frac{1}{2} Q + 8 \right)^2 = (2\sqrt{Q - 1})^2
\]

Be careful when we square both sides:

\[
\left( -\frac{1}{2} Q + 8 \right)\left( -\frac{1}{2} Q + 8 \right) = (2\sqrt{Q - 1})(2\sqrt{Q - 1})
\]

\[
\frac{1}{4} Q^2 - 8Q + 64 = 4(Q - 1)
\]

\[
\frac{1}{4} Q^2 - 8Q + 64 = 4Q - 4
\]

Next, set the right side of the equation equal to zero by subtracting \( 4Q \) from both sides and adding 4 to both sides:

\[
\frac{1}{4} Q^2 - 12Q + 68 = 0
\]

Solve using the quadratic formula for both the plus and minus sign. Keep your answer to the nearest hundredth.
From past experience, please recall that this is a quadratic equation of the form \( ax^2 + bx + c = 0 \). In this case we have

\[
\begin{align*}
    a &= \frac{1}{4} \\
    b &= -12 \\
    c &= 68
\end{align*}
\]

In order to solve this equation, we need to use the quadratic formula

\[
Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Substituting \( a, b \) and \( c \) into the quadratic formula, we have

\[
Q = \frac{-(-12) \pm \sqrt{(-12)^2 - 4\left(\frac{1}{4}\right)(68)}}{2\left(\frac{1}{4}\right)}
\]

Using the plus sign the answer is \( Q = 41.44 \)

Using the minus sign the answer is \( Q = 6.56 \)

From the graphs above we recognize that \( Q = 6.56 \) is the answer we are looking for. But where does the other answer come from and why did it appear.

Let’s put \( Q = 41.44 \) back into both the supply and demand equations and solve for the price of each equation to the nearest hundredth.

Supply: \( P_s = 2\sqrt{41.44} - 1 + 3 = 9.72 \)

Demand: \( P_d = -\frac{1}{2}(41.44) + 11 = -9.72 \)

Above, we said that we would return to the point in our work where we squared both sides. Realize that we set the price values of the supply and demand curve equal to each other and then squared both sides. So while \( 9.72 \) does not equal \( -9.72 \), if we square both numbers we do get the same answer, \( (9.72)^2 \) does equal \( (-9.72)^2 \). Thus the process of completing the squares has created an **extraneous solution**. By extraneous solution, we mean a solution which is extra and false. Thus, when working problems that require us to square both sides, we must be careful to check our answers for extraneous
solutions. In this case we do so by placing the answer in the problem and checking to make sure that the prices are the same including the sign.

If you are on the lookout for situations where extraneous solutions appear, you will be able to add solving problems using exponents in the very near future to any situation where you square both sides.

For the following problems follow the above steps to find the equilibrium points for the supply and demand equations. Make sure to remove extraneous solutions. If decimal answers appear, round to the nearest hundredth.

1) \( P_s = 3\sqrt{Q_s - 4} + 5 \)
   \( P_d = -3Q_d + 26 \)

2) \( P_s = 4\sqrt{Q_s - 2} + 5 \)
   \( P_d = -4Q_d + 21 \)

3) \( P_s = 2\sqrt{Q_s - 2} + 5 \)
   \( P_d = -Q_d + 21 \)

For the following problems follow the above steps to find the Price which creates equilibrium for the supply and demand equations. Make sure to remove extraneous solutions. If decimal answers appear, round to the nearest hundredth.

4) \( P_s = 2\sqrt{Q_s - 3} + 7 \)
   \( P_d = -2Q_d + 26 \)
5) Supply: $P_s = \sqrt{Q_s - 2} + 5$
Demand: $P_d = -2Q_d + 21$

6) Supply: $P_s = 2\sqrt{Q_s - 2} + 5$
Demand: $P_d = -2Q_d + 21$

For the following problems follow the above steps to find the quantity which creates equilibrium for the supply and demand equations. Make sure to remove extraneous solutions (Note: in this case, you will still need to find the price in order to determine the extraneous solution). If decimal answers appear, round to the nearest hundredth.

1) Supply: $P_s = 2\sqrt{Q_s - 2} + 8$
Demand: $P_d = -2Q_d + 26$

2) Supply: $P_s = \sqrt{Q_s - 2} + 5$
Demand: $P_d = -2Q_d + 21$

3) Supply: $P_s = 2x - 3$
Demand: $P_d = -x + 7$