Last week we looked at momentum problems, \( p = mv \). Consider the following problem:

If a 3000kg truck traveling at 5 m/s strikes a 2000 kg sedan which is stopped at a stop light and if the two vehicles stick together (moving as one) what is the speed of the two vehicles after the collision? Ignore friction.

The law of Conservation of linear momentum states that the sum of the momentum of the two vehicles before the crash is equal to the momentum of the two vehicles after the crash.

To accomplish this problem, we will use subscripts to distinguish between the truck \((t)\) and the sedan \((s)\) before the crash and the combined vehicles \((c)\) after the crash. Hence are variables are:

- \( p_c \) – Momentum of the combined vehicles after the crash
- \( p_t \) – Momentum of the truck before the crash
- \( p_s \) – Momentum of the sedan before the crash
- \( m_c \) – Mass of the combined vehicles = 3000 kg + 2000 kg = 5000 kg
- \( m_t \) – Mass of the truck = 3000 kg
- \( m_s \) – Mass of the sedan = 2000 kg
- \( v_c \) = Speed of the combined vehicles after the crash (What we are looking for)
- \( v_t \) – Speed of the truck = 5 m/s
- \( v_s \) – Speed of the sedan = 0 m/s

This problem has more letter representations than most at the math 108 level which is why we need the subscript to assist us in distinguishing the variables while at the same time still allowing the letters to represent momentum, mass and velocity. When working a word problem the first step is to list the symbols that will be used and if you know the values to list them. Thus, we have completed the first step to solve this problem.

The second step is to identify what we are looking for, which we have also completed – Combined Velocity - \( v_c \). Next we identify the equations we know using \( v_c \) (and we insert known values where possible):

\[
\begin{align*}
p_c &= m_c v_c \\
p_c &= 5000v_c
\end{align*}
\]

From this equation, we can determine \( v_c \), if we know \( p_c \). Thus, let us look for equations that might help us find \( p_c \). Based on the law of conservation of momentum, we know that

\[
p_c = p_t + p_s
\]
From last week, you should be able to find both $p_r$ and $p_s$. Thus, we will allow you to finish the problem from here and determine $v_c$.

We have used the word crash in the problem to help visualize the move into forensic science and the mathematics needed to develop an understanding of an accident scene by only looking at the end results of the accident. But this problem also shows the difficulty of attempting to provide examples of applications of mathematics at the math 108 level. If we arrived on a true accident we would need more algebra skills, trigonometry skills, calculus skills and quite possibly some additional skills as we would need to include factors such as the direction the vehicles are moving, the speed of each vehicle, the directions the two vehicles are moving after the crash, changes in elevation, friction from the road (or ground) and how the cars absorb the energy through the process of being crushed.

In an effort to provide students with problems at the math 108 level, problems are often brought down to the level that they no longer represent real world situations (such as the problem above). For example we might find one of the most misunderstood mathematical problems of them all:

\[
\text{Airplane A left the airport heading due west at noon traveling 250 miles per hour and Airplane B left the airport at 1:00 traveling 300 miles per hour. At what time will the two airplanes be 1000 miles apart?}
\]

Realize that the mathematics we are working on today is the beginning basis of the mathematics required for mixture problems, evaluating chemical equations, understanding interactions of species in biology, moving objects such as that found in civil engineering, physics (forces) and atom smashing.

Take some time to discuss this situation in your groups. Why would we ask the students to complete problems that might indicate some usefulness of mathematics but are not really useful in and of themselves? Jot a few quality reasons on the page below:
Now that you have listed your reasons, let me explain one of the possible answers. Our introductory problem of the two vehicles colliding involved nine variables and four different equations to arrive at the answer. Furthermore, if we desired to allow all of the real conditions into the problem, we would need to drastically increase the number of variables and we would need to increase both the number of equations and the complexity of the equations. If students waited until problems of this complexity appeared before building their problem solving skills, they would be overwhelmed by the process. Hence, we develop simplified problems which have only two variables and two equations required to solve the problem and as we move through mathematics and the students’ skills increase, we will increase the complexity of the problems.

Hence, the most important aspect of these problems is to develop the following skills:

1) Read the entire problem several times to get a complete understanding.
2) Define all of the variables in the problem.
3) Determine the equations that are useful.
4) Solve the problem
5) Check your answer to make sure it is reasonable.
6) State your answer in a sentence.

During today’s activity, your instructor will be more concerned that you learn the correct process to solve the problems than anything else. It is these processes that will make more complicated problems easier.

Example 1: Mixture Problem: A chemist has a 20% alcohol solution and a 60% alcohol solution to create 60 ml of a 55% solution. How much of each solution is needed to form the desired results?

1) Define the variables. In this problem we have two quantities that are not known. The amount of solution needed that is 20% alcohol and the amount of solution that is 60% alcohol. In this case there is not an easy reference letter so we will use $x$ and $y$ as our variables, providing the following defined variables:

$x = \text{the number of ml of 20\% solution needed}$

$y = \text{the number of ml of 60\% solution needed}$

2) Read carefully and we see that the combined solution is 60 ml. This makes the first equation very simple:

$x + y = 60$

The second equation has to do with the percent alcohol and the number of ml of each item. Since the final amount is 60 ml of 55% alcohol, we have $0.55 \times 60 = 33 \text{ ml of alcohol}$. This multiplication can be created with each individual amount and when combined should be equal to the mixture giving an equation of

$0.20x + 0.60y = (0.55)(60)$
3) We now have two equations. Use the two equations to solve the problem

\[
x + y = 60
\]
\[
0.20x + 0.60y = 33
\]

4) Check your answer to make sure it makes sense. In this problem the easy method is to make sure the two answers do add up to 60.

Example 2: A boat traveled up river 24 miles in 3 hours. It returned in 2 hours. What is the speed of the boat and what is the speed of the rivers current?

1) There are two unknowns:

\[
b = \text{the speed of the boat}
\]
\[
c = \text{the speed of the rivers current}
\]

2) In this problem, the primary equation is distance = rate x time \((d = rt)\). From the wording above you should be able to figure out the distance and the time for both directions to obtain two equations. However, the rate is \(b - c\) going upstream and \(b + c\) going downstream. Write the two equations in terms of \(b\) and \(c\).

3) Use your two equations to solve the problem

4) Check your answer (does it make sense). No answer should be negative.

Example 3: Introduction to nutrition. Food source A has fifty calories and three grams of protein per ounce. Food source B has forty calories and six grams of protein per ounce. How many ounces of each food must be eaten so that the patient has 640 calories and 50 grams of protein?

1) The variables are easy

\[
A = \text{the number of onces of food source A}
\]
\[
B = \text{the number of onces of food source B}
\]

2) The first equation relates the calories from each item with the total number of calories

\[
50A + 40B = 640
\]

The second equation relates the grams of protein in the same fashion. Find the second equation.
3) Solve the system of equations.

4) Does the answer make sense?

In the first two examples, these types of simple problems actually are used on a regular basis. The third example shows how a diet can be monitored. Usually the number of items concerned with is much greater and are equations or inequalities, making them more difficult, but something we should be able to answer by the time we complete college algebra.

Here are some practice problems:

1) Mixture Problem: A chemist has wants to combine a 30% alcohol solution with a 45% alcohol solution to create 800 ml of a 40% solution. How much of each solution is needed to form the desired results?

2) Mixture Problem: A chemist has wants to combine a 10% alcohol solution with a 55% alcohol solution to create 90 ml of a 30% solution. How much of each solution is needed to form the desired results?

3) Mixture Problem: A chemist has wants to combine a 20% alcohol solution with a 60% alcohol solution to create 700 ml of a 35% solution. How much of each solution is needed to form the desired results?
1) A boat traveled up river 60 miles in 3 hours. It returned in 2 hours. What is the speed of the boat and what is the speed of the rivers current?

2) A boat traveled up river 80 miles in 5 hours. It returned in 4 hours. What is the speed of the boat and what is the speed of the rivers current?

3) A boat traveled up river 100 miles in 10 hours. It returned in 8 hours. What is the speed of the boat and what is the speed of the rivers current?

1) Introduction to nutrition. Food source A has 65 calories and 5 grams of protein per ounce. Food source B has 22 calories and 7 grams of protein per ounce. How many ounces of each food must be eaten to have so that the patient has 654 calories and 114 grams of protein?

2) Introduction to nutrition. Food source A has 13 calories and 7 grams of protein per ounce. Food source B has 19 calories and 11 grams of protein per ounce. How many ounces of each food must be eaten to have so that the patient has 238 calories and 132 grams of protein?