Supply and demand is a fundamental concept in business.

**Demand** looks at the Quantity (Q) of a product that will be sold with respect to the Price (P) the product can be sold for. Realize that both quantity (Q) and price (P) can be represented by numbers and this implies the use of mathematics. Furthermore, with a little logical reasoning, we can realize that there is a relationship between quantity (Q) and price (P). Consider the two situations:

1) If I am a store manager with a certain product that I have too many of and I want the shelf space for some other item, what might I do to the price?
2) If I am a store manager and there is a product that has so much demand that it never stays on the shelf, what might I do to the price?

To develop a model of demand we will place quantity (Q) on the horizontal axis and price (P) on the vertical axis. We have been writing using a (Q) and a (P) after quantity and price to assist us in moving to using Q for the variable to represent quantity and P for the variable to represent the price. Thus, our points will be in the form of (Q, P). With \( m \) representing slope and \( b \) representing the vertical intercept, the liner demand equation will be of the form:

\[
\text{Demand: } P = mQ + b
\]

This should be recognized as the slope-intercept form of a line \( y = mx + b \).

Someone has completed a market research analysis which indicates that when the quantity is 2 units, the price determined by demand will be 10. And when the quantity is 12 units, the price determined by demand will be 5.

Write the two points which are represented from the above statement (remember to make sure your two points are in the correct order \( (Q, P) \)).

Point 1: \((__, __)\)

Point 2: \((__, __)\)

Using these two points graph the demand curve.
Find $m$ and $b$ so that we know the equation of the demand model. Also write down the equation.

Discuss the implied domain (feasible region). Realize that negative quantity and negative price really do not make sense in this problem. Thus the two intercepts provide reasonable limitation for the implied domain (feasible region).

What is the implied domain?

**Supply** looks at the Quantity ($Q$) of a product that producers will produce with respect to the Price ($P$) the product can be sold for. For supply there is also a relationship between quantity ($Q$) and price ($P$). In the case of supply, if we increase supply, we will also need to increase price.

As in our work with demand, the quantity will be on the horizontal axis and the price will be on the vertical axis. With $m$ representing slope and $b$ representing the vertical intercept, the linear demand equation will be of the form:

$$\text{Supply: } P = mQ + b$$

At this point one should have a problem as the supply and the demand equation are identical. One method used by some textbooks to distinguish the two equations is to use a subscript $d$ and a subscript $s$ to determine whether the quantity and price is demand or supply. In this fashion a demand point would be in the form: $(Q_d, P_d)$. The supply point would be in the form $(Q_s, P_s)$. This would lead to the following equations:

$$\text{Demand: } P_d = mQ_d + b$$

$$\text{Supply: } P_s = mQ_s + b$$

Whether you use this method or another strategy, you will need to recognize that both $Q$ and $P$ are used in the supply and demand equations with a slightly different meaning.
In the same market research analysis completed above, it was indicated that when the quantity is 2 units, the price determined by supply will be 1. And when the quantity is 12 units, the price determined by supply will be 11.

Write the two points which are represented from the above statement (remember to make sure your two points are in the correct order \((Q, P)\).

Point 1: \((__, __)\)

Point 2: \((__, __)\)

Using these two points graph the supply curve.

Find \(m\) and \(b\) so that we know the equation of the demand model. Also write down the equation.

Determine the implied domain (feasible region). The lower end point will occur at the horizontal intercept. The upper bound is fuzzy. Explain why you choose that bound.
Putting Supply and Demand together:

Now that we have both a demand and a supply curve, we can put them together on one graph. To do so, let’s first find the implied domain (feasible region) of the supply and demand system. Consider:

1) It is easy to see that the parts of an automobile are clearly more expensive than $10.00. Thus, there would be no supply of automobiles if the price of the automobile drops below the cost to manufacture the vehicle. There is no reason to talk about demand at this level. Thus, the lower end of the feasible region is the lower end of the feasible region for the supply curve.

2) At the upper end, there becomes a price where the vehicle becomes too expensive for anyone to desire to purchase the vehicle. Thus, the upper end of the feasible region is determined by the upper end of the demand curve.

We have found the implied domain (feasible region) of the problem by logically resolving the issue. To solve the problem from a mathematical standpoint, we would write down the implied domain of both the demand function and the implied domain of the supply function and find the intersection.

Answer = [___, ____] \( \cap \) [___, ____]

Demand region \( \cap \) Supply region

What is the implied domain of the supply and demand curve?

Graph the supply and demand curves on the same graph. Note: Since all values for both the supply and demand curve are positive in the implied domain (feasible region), we will graph only in the first quadrant of the graph:
Notice that the two curves intersect. Often in math we ask students to find the point of intersection. However, in business this point is called the equilibrium point.

Find the equilibrium point.

What is the quantity needed for equilibrium?

What is the price at which equilibrium is reached?

For the following system of supply and demand equations, find the equilibrium point.

1) Demand: \( P_d = -2Q_d + 28 \)
   Supply: \( P_s = 4Q_s + 1 \)

2) Demand: \( P_d = -3Q_d + 60 \)
   Supply: \( P_s = 2Q_s - 6 \)

3) Demand: \( P_d = -2Q_d + 28 \)
   Supply: \( P_s = 4Q_s + 1 \)
For the following system of supply and demand, find the price at equilibrium. (Notice that these problems are very similar to the last problems. The difference is that you are being asked to find an answer that is just one value of the equilibrium point. Many students believe that they should be given credit for this type of problem if they just find the equilibrium point however, the design of the problem really is important and giving the wrong answer is not correct.)

4) Demand: \( P_d = -2Q_d + 28 \)
   Supply: \( P_s = 4Q_s + 1 \)

5) Demand: \( P_d = -3Q_d + 39 \)
   Supply: \( P_s = 2Q_s - 12 \)

6) Demand: \( P_d = -7Q_d + 77 \)
   Supply: \( P_s = 14Q_s + 3 \)
For the following systems of supply and demand, find the quantity which will lead to equilibrium.

7) Demand: \( P_d = -2Q_d + 28 \)
   Supply: \( P_s = 4Q_s + 1 \)

8) Demand: \( P_d = -Q_d + 35 \)
   Supply: \( P_s = 2Q_s - 12 \)

9) Demand: \( P_d = -7Q_d + 77 \)
   Supply: \( P_s = 14Q_s + 10 \)