**Mathematical modeling** is simply the act of building a model (usually in the form of graphs) which provides a “picture” of a numerical situation.

**Linear modeling** is mathematical modeling where the graph is a line, hence linear. Linear modeling occurs any time the rate of change is constant. Let’s build two examples.

**Example 1:**

In physics, momentum is linearly associated with velocity of an object. Hence, we will want to build a linear model. To begin to build our linear model, let’s pick two points.

For the first point when an auto is traveling 5 meters per second (about 11 miles per hour), the momentum of the auto is 7500 kg m/s. (Note: kg m/s is just shorthand for the units of measurement of momentum which is kilograms times meters divided by seconds.)

Enter the answer in coordinate form: (___, ___) (Answer in coordinate form: (5, 7500))

For the second point, when the auto is traveling 25 meters per second, the momentum is 37,500 kg m/s.

Enter the answer in coordinate form: (___, ___)

Now that we have two points, let’s create a graph. As we start graphing our two points, we draw and label two axes. But, what should we use as labels? What letter should we use to represent our variables? Since we currently are in a math course, it would be easy to choose x and y as our representative letters. But what do x and y mean? Using the letters, x and y, in math makes perfect sense as we are able to move from one mathematical concept to the next with a strong level of consistency and thus, math is easier to learn from chapter to chapter and course to course.

When we leave math and move to other subjects, such as physics, we will choose our letters which reflect the situation we are modeling. In this case, physicists use v to represent velocity and p to represent the momentum. In this case, a general point will be in the form (v, p).

**Side note:** Many students have difficulty when we use letters other than x and y. When students enroll in other courses which use mathematical modeling and do not use x and y as the variable, they often have difficulty relating their other course to mathematics. In this case the new material is more difficult to learn.

Hence, the sooner the student recognizes that an equation using letters is math regardless of the letters being used, the easier their overall education will be.
The variable \( v \), representing velocity, is called the independent variable and will be placed on the horizontal axis when graphing.

The variable \( p \), representing momentum, is called the dependent variable. The vertical axis will be used to represent \( p \).

Having two points and knowing that this problem is represented by a linear model, the graph can be created. Use the set of axes below to draw the graph. Please label the \( v \) and \( p \) axes respectively. Be careful to think about the scale of the problem before beginning to place the points.

Now that the graph has been created, the graph can be used to estimate the momentum of the auto for any given velocity. If we look at the graph and consider the scale, we recognize that the best the graph can do is a rough estimation. If the model is needed to have more accurate estimations, then we need to create a numerical solution for the graph. Thus, we need to create the equation of the line. This will require finding both the slope, \( m \), and the \( y \)-intercept, \( b \).

Finding the slope: To find the slope we use the equation

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

One might recognize that in this problem we no longer have an \( x \) or a \( y \) and be confused. But this is easy to overcome when we realize that all we did was replace \( x \) with \( v \) and \( y \) with \( p \), giving the equation as
Compute the slope.

Next we need to find the $y$-intercept. Recall that we changed the variable of $y$ to $p$. Hence we need to find the $p$-intercept. The actual intercept that we are looking for to complete the equation of the line is the intercept of the vertical axis. There are mathematical processes which can be used to find this value. This equation is called the point slope. Using our new variables, the point slope equation is

$$p - p_1 = m(v - v_1)$$

Rewrite the equation replacing the slope, $m$, and solving for the slope intercept form of the equation,

$$p = mv + b$$

The equation is ____________________________

In your equation the $p$-intercept (value of $b$) is_____. Explain why this makes sense:

The general form of a linear momentum equation is:

$$p = mv$$

Please realize that this set of letters to represent unknowns was chosen by physicists. As stated before $p$ represents momentum and $v$ represents velocity. So $m$ represents???

That is right, $m$ represents mass and in this case the mass of the auto is 1500 kg. In mathematics the slope generally means the steepness of the line. However, outside mathematics, when you are using linear equations, the slope generally has a far more important meaning as in this case, where it represents the mass of the auto.

Finding the implied domain: There is one final step to completing this example and this step is to find the implied domain. Implied domain is the correct mathematical term but, outside of math, terms such as feasible region will be used.

Recall that domain represents the set of all possible $x$-values (sometimes referred to as input values). In the case of our example, the domain is the set of all $v$-values. Based on the equation written above, there is no restriction on the domain so the domain is all real numbers.

This problem has more to it than just an equation. It has a given set of conditions which allowed us to write an equation. What restrictions are put on the problems based on the conditions of velocity? This restriction is called the implied domain (or feasible region).
On the lower side, one can easily find a boundary where the problem no longer makes sense. Since we are not concerned about the direction the car is headed, the velocity must be greater than or equal to zero. Hence, we have $v \geq 0$.

Now for the right hand side: What is the maximum speed of the car? We haven’t discussed the type of car. Is it a fuel economy car with a maximum speed of 90 miles per hour? Or is it a NASCAR racing machine with a maximum speed of 250 miles per hour? What is the upper bound? On the right hand side, the feasible region is not limited by a single number and different groups will produce different values for the upper limit. In your group, decide on an upper limit. Write the feasible region (implied domain) and draw a new graph. Note, make sure your answer is in meters per second and not miles per hour.

But wait a moment, don’t cars have reverse? Couldn’t we then say that there is a negative velocity? Rethink your implied domain and give a best implied domain. Remember that maximum reverse speed is less than maximum forward speed.

End of Example 1.

Example 2:

Since we are discussing linear momentum we should recognize that one point of the form $(v, p)$ will be $(0, 0)$ for every equation. Given that a vehicle has a maximum velocity of 8700 m/s and the momentum is 913,500,000 kg m/s. Write down the second point. Determine the equation of the line and graph the equation.
Recall that slope, represents the mass of the vehicle in kgs. What is the mass of the vehicle? What is the vehicle?

Discuss the feasible region (write a brief answer describing your thoughts on the lower and upper bound).

End of Example 2.

Here are three more linear momentum problems. In each case find the linear equation and using the mass of the vehicle try to guess the vehicle being used in the model.

1) \((0, 0)\) and \((20, 3360)\)
2) \((0, 0)\) and \((15; 540,000)\)
3) \((0, 0)\) and \((8, 29600)\)
Here is some more practice. Find the equation of the line and graph.

1) \((0, -10)\) and \((5, 25)\)
2) \((0, 15)\) and \((5, 7)\)
3) \((6, 3)\) and \((5, 7)\)
4) \((8, 13)\) and \((10, 7)\)
5) \((-5, 9)\) and \((4, 8)\)
6) \((-2, -4)\) and \((-8, -7)\)