In this activity, we are going to look at modeling business profits. We will allow \( q \) to represent the number of items manufactured and assume that all items that are manufactured are also sold. To simplify matters, rather than discussing generic items being manufactured, let’s assume we are manufacturing graphing calculators. Sometime later we will need to know what the capacity of our ability to manufacture calculators is. In this case we will say that we have the ability to build at most 15,000 calculators.

We are interested in finding the profits as a function of the quantity manufactured, so we will label profits as \( P(q) \). Two other terms that are important in profit are cost and revenue:

- **Cost** is the amount of money spent to manufacture \( q \) items. Thus, we will denote cost as \( C(q) \).
- **Revenue** is the amount of money earned by selling \( q \) items. Thus, we will denote revenue as \( R(q) \).

The importance of the cost and revenue functions is that they can be used to compute the profits function which is what we are interested in. The profit function can be found by computing:

\[
P(q) = R(q) - C(q)
\]

Thus, to find the profit function, it will be easier to find both the revenue function and the cost function.

**Finding the cost function:** In a simple model the cost function is based on two parts:

- **Fixed costs:** The costs that will be spent whether or not a single item is built such as mortgage payments on the building, property taxes, and electricity. In this example, let’s say that the fixed costs are $20,000.00.
- **Item costs:** The cost required to manufacture a single item including the cost of raw material and labor. In this case, let’s say it costs $40.00 to manufacture each calculator. Note: Since it costs $40.00 per calculator, the total cost of manufacturing a total of \( q \) calculators is $40.00 \times q.

The cost equation is: \( C(q) = \) _____________.

**Finding the revenue function:** Revenue is computed by the simple formula: (price per item)X(quantity sold). Quantity sold is simply represented by ‘\( q \)’. In finding the price per item, we have to recall our work on the supply and demand. Based on what we learned about demand, as the quantity of calculators sold is increased the price must decrease. Hence we need a demand equation to figure out the price per item. In this example we will use the following demand equation.

\[
\text{Demand: } p = -0.004q + 95.00
\]
Looking at the demand equation, determine what the price per item should be to sell 10,000 calculators.

The price per item would be = ____________________

What is the price per item when one item is sold?

If we graph the problem, what does the vertical axis intercept represent?

What is the slope? ______ and what does it imply?

Describe what the demand equation is modeling:

Returning to the revenue equation, we can find revenue by multiplying \( p \times q \) (price times quantity). In finding revenue, we desire to use the demand equation instead of simply using \( p \).

The revenue equation is: \( R(q) = \) ____________________________.

Simplify the revenue equation:

With both the revenue equation and the cost equation, we can now find the profit equation.

\[
P(x) = R(x) - C(x)
\]

Because of the minus sign and because both the revenue function and the cost function have two (or more) terms when placing these into the profit function, be sure to use parentheses. Thus

\[
P(x) = (_______________) - (_______________)
\]

Simplify the profit equation.
Before we graph the profit equation, let's rethink the problem and find the implied domain (feasible region). The implied domain will be the limitations on the problem based on the quantity, $q$. What is the smallest number of calculators that can be manufactured and what is the largest number of calculators that can be manufactured? The implied domain is:

Graphing the profit equation, we obtain:

We have plotted five points:

- The vertex, where maximum profit is indicated.
- Two $q$-intercepts or break even points
- The two endpoints of our implied domain.

The vertex, which indicates the maximum profit, is important, so let’s compute that value. From our information about quadratics equations, we know that given the polynomial is:

$$p(x) = ax^2 + bx + c$$
That the x-value of the vertex is:

\[ x = -\frac{b}{2a} \]

In our specific case, we have a profit equation:

\[ P(q) = aq^2 + bq + c \]

And we desire to find the quantity at the maximum profit (or vertex). This can be found in the same fashion by finding:

\[ q = -\frac{b}{2a} \]

Once we have the value for \( q \), we can insert this result into the profit equation to find the maximum profit.

Find the maximum point: (Both the quantity for creating maximum profit and the maximum profit.)

The break-even points are called break-even points, because the profit is zero at these points. Also as you move from left to right through these points, you will move from losing money to making money or vice versa. To find the break-even points with this model, we simply use the quadratic formula.

\[ q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Find the break-even points (remember to round to the nearest unit).

Maximum quantity: \( q = \frac{-b}{2a} \)

Break – even quantity: \( q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

For definition purposes, realize that the value inside the square root sign, \( b^2 - 4ac \), is called the discriminant. We discussed this value last week.
Below, please discuss how the two quantities are similar, and relate the discriminant to the graph:

Find the quantity needed for maximum profit of the following profit equations:

1) \( P(q) = -.001q^2 + 100q - 50,000 \)

2) \( P(q) = -.002q^2 + 70q - 10,000 \)

Find the maximum profit of the following profit equations:

3) \( P(q) = -.02q^2 + 1000q - 5,000 \)

4) \( P(q) = -.05q^2 + 10000q - 50,000 \)

Find the maximum point of the following profit equations:

5) \( P(q) = -.001q^2 + 1000q - 50,000 \)
6) \( P(q) = -.003q^2 + 100q - 50,000 \)

Find the break-even points for the following profit equations and describe the region where positive profits are created.

7) \( P(q) = -.001q^2 + 100q - 50,000 \)

8) \( P(q) = -.002q^2 + 70q - 10,000 \)

9) \( P(q) = -.02q^2 + 1000q - 5,000 \)

10) \( P(q) = -.05q^2 + 10000q - 50,000 \)

11) \( P(q) = -.001q^2 + 1000q - 50,000 \)

12) \( P(q) = -.003q^2 + 100q - 50,000 \)