Math 144
Activity #8
Solving Trigonometric Equations

In a previous activity, you looked at evaluating expressions where you were taking the inverses of trigonometric functions. You had to first make the original trig function one to one and then you were able to take the inverse. This week, you are going to be evaluating inverse trig expressions when you are solving trig equations.

Remember that taking the inverse of a function, undoes a function much like adding undoes subtracting, multiplication undoes division etc.

Let's recall what inverse functions are: $f\left(f^{-1}(x)\right)=x$ and $f^{-1}\left(f(x)\right)=x$ this means that functions that are inverses of each other are functions that undo each other. This works for trig functions as well.

trig func (angle) = ratio for example $\sin\theta=\frac{a}{b}$ or $\cos\theta=\frac{c}{d}$ etc.

inverse trig func (ratio) = angle for example $\sin^{-1}\frac{a}{b}=\theta$ or $\cos^{-1}\frac{c}{d}=\theta$

What does solving an equation mean both graphically and algebraically?

What does solving the equation $3\sin\theta-1=-1$ on the interval $[0,2\pi)$ mean both graphically and algebraically?

What does finding all solutions to the equation $3\sin\theta-1=-1$ mean both graphically and algebraically?

First, solve the equation graphically on the interval $[0,2\pi)$, in 2 different ways. Sketch the graph and indicate your solutions on the graph.

Now, solve the equation $3\sin\theta - 1 = -1$ algebraically over the interval $[0, 2\pi)$.

If we did not limit the interval to $[0, 2\pi)$, how many solutions would there be? Explain. To help you with this, look at how often the solutions occur.

Look at the eight equations below, what are some of the similarities and differences between them?

1. $2\cos\theta - 1 = 0$
2. $8\sin\theta - 4\sqrt{3} = 0$
3. $\tan^2 x - 1 = 0$
4. $4\cos^2 x = 5 - 4\sin x$
5. $\sin^2 x - 5\cos x = 5$
6. $4\csc^2 x - 3 = 0$
7. $\tan^2 x - \tan x = 0$
8. $\left(\sqrt{3}\sec x + 2\right)\left(\csc x - \frac{1}{4}\right) = 0$

Solve each equation over the interval $[0, 2\pi)$.

1. $8\sin x - 4\sqrt{3} = 0$
2. $4\cos^2 x - 3 = 0$

3. $\tan^2 x - 1 = 0$

4. $4\cos^2 x = 5 - 4\sin x$

5. $\tan^2 x - \tan x = 0$

6. $\left(\sqrt{3}\sec x + 2\right)\left(\csc x - \frac{1}{4}\right) = 0$

Solve each equation. Give a general solution.

1. $2\cos\theta - 1 = 0$

2. $2\cos x - \sqrt{2} = 0$

3. $\cos\theta - 1 = 0$

4. $\sec\theta + \sqrt{2} = 0$

5. $\sqrt{3}\csc\theta - 2 = 0$

6. $4\sin^2 x - 2 = 0$

7. $\tan x + \sqrt{3} = 0$

8. $3\tan^2 x - 1 = 0$

9. $\cot^2 x - \cot x = 0$

10. $\cos 3x = 0$

11. $\sec\left(\frac{3x}{2}\right) + 2 = 0$

12. $\sin^2 x - 4\sin x - 5 = 0$

13. $\cos 2x - \sqrt{3}\cos x = -1$

14. $\cot x \sec x + \cot x = 0$

Solve each equation over the interval $(-2\pi, 2\pi)$.

13. $16\cos^2 x - 8 = 0$

14. $2\cos^2 x - 3\cos x + 1 = 0$

Use a graphing calculator to solve the equation. For full credit, sketch the graph. Indicate where your solution lies on the graph. Round to the nearest hundredth if necessary.

$$\sin 2x = -0.4x + 0.9$$