Math 144
Activity #6
Inverse Functions

In this activity we will discuss characteristics of functions and their inverses. We will talk about functions and their inverses both graphically and algebraically.

Functions can be represented as graphs, equations, ordered pairs or descriptions. So first we must ask the question—what makes a relation a function? Write down a concise definition for function. Make sure to talk about domain, range, graphs and equations. In your groups, come up with a “best” definition.

By your definition are the six trig functions really functions? Explain each one separately.

This semester we have already agreed that sine, cosine, tangents, secants, cosecants and cotangents are functions. What makes them functions? When you answer this question list the domains and ranges for each of the six functions.

Now we want to discuss if they have inverses. What is the definition of an inverse? Answer both graphically and algebraically.

Do the 6 trig functions have inverses? Is so, why? If not, why not?

Let’s focus on the graph of $y = \sin x$. Looking at the sine graph, does it have an inverse? If it does NOT have an inverse, how can we make it have an inverse?
In order to have an inverse, we need to restrict the domain. We restrict the domain to \([-\frac{\pi}{2}, \frac{\pi}{2}]\) thus now it has an inverse and is written as arcsine or \(\sin^{-1}\) (read “sine inverse”). (Note: \(\sin^{-1}\) is NOT \(\sin\) raised to the negative 1 power.) Explain where the restriction appears on the unit circle. That is, where is \(\sin^{-1}\) defined on the unit circle?

Explain where the restriction appears on the sine graph.

Looking at the cosine graph, does it have an inverse? If not, how can we make it have an inverse?

In order to have an inverse, we need to restrict the domain to \([0, \pi]\) thus now it has an inverse and is written as arccosine or \(\cos^{-1}\). Where does this restriction appear on the unit circle? That is where is \(\cos^{-1}\) defined on the unit circle? Where does the restriction appear on the cosine graph?

Looking at the tangent graph, does it have an inverse? If not, how can we make it have an inverse?

In order to have an inverse, we need to restrict the domain to \((-\frac{\pi}{2}, \frac{\pi}{2})\) thus now it has an inverse and is written as arctan or \(\tan^{-1}\). Note where the restriction appears on the unit circle and on the tangent graph.
Looking at the graph of \( f(x) = \sin x \). Sketch the graph of \( f(x) = \arcsin x \) after you have restricted the domain. What are the domains and ranges of both \( \sin x \) and \( \arcsin x \)?

Looking at the graph of \( f(x) = \cos x \). Sketch the graph of \( f(x) = \arccos x \) after you have restricted the domain. What are the domains and ranges of both \( \cos x \) and \( \arccos x \)?

Some inverse function questions:

1. What is the angle whose cosine is \( \frac{1}{2} \)? Explain how you found this. Are there other angles whose cosine is \( \frac{1}{2} \)? Explain.

2. What is the angle whose sine is \( \frac{1}{2} \)? Are there other angles whose sine is \( \frac{1}{2} \)?

3. If \( y = \sin x \) is the inverse of \( y = \arcsin (x) \), what is \( \sin (\arcsin (x)) \)? Explain. What is \( \arcsin (\sin (x)) \)? Can you find any examples where these compositions would NOT work? If so, give them. If not, explain.

4. Find the exact value of \( \cos^{-1}\left(\frac{1}{2}\right) \). When you are finding \( \cos^{-1}\left(\frac{1}{2}\right) \) what exactly are you finding? An angle measure, a ratio? Locate the solution on the unit circle and on the graph of \( y = \cos x \).

5. Find the exact value of \( \arccos\left(-\frac{\sqrt{3}}{2}\right) \). When you are finding \( \arccos\left(-\frac{\sqrt{3}}{2}\right) \) what exactly are you finding? An angle measure, a ratio? Locate the solution on the unit circle and on the graph of \( y = \cos x \).

6. Find the exact value of \( \tan^{-1}\left(\tan\frac{3\pi}{4}\right) \). Locate the solution on the unit circle and on the graph of the \( y = \tan x \).
7. Find the exact value of \( \tan^{-1}\left(\tan\frac{5\pi}{3}\right) \).

8. Find the exact value of \( \sec\left(\arcsin\left(\frac{7}{10}\right)\right) \).

9. Find the exact value of \( \sin\left(\tan^{-1}\left(-\frac{12}{5}\right)\right) \).

10. Find the exact value of \( \arcsin\left(\cos\frac{2\pi}{3}\right) \).

11. Find the exact value of \( \tan^{-1}\left(\tan\frac{5\pi}{3}\right) \).

12. Find the exact value of \( \cos(\arctan(1)) \).

13. Rewrite \( \sin(\tan^{-1} v) \) as an algebraic expression in \( v \).

14. Rewrite \( \cos\left(\tan^{-1}\frac{3v}{5}\right) \) as an algebraic expression in \( v \).

15. Rewrite \( \cot\left(\cos^{-1}\frac{5v}{9}\right) \) as an algebraic expression in \( v \).