Applications of Exponential Functions
Group Activity 7
STEM Project Week #10

In the last activity we looked at exponential functions. We looked at an example of a population growing at a certain rate. In this activity, we will look at a substance decreasing or decaying at a certain rate.

The data set
The half-life of radium-222 is 38 seconds. This means that every 38 seconds the amount of radium 222 is cut in half. Suppose we start with 144 grams of radium-222. Make a table of values where the amount, $N(t)$ is your y-value and the time, $t$ is your x-value. Find at least 5 points on your graph.

<table>
<thead>
<tr>
<th>$t$</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(t)$</td>
<td>amount</td>
</tr>
</tbody>
</table>

Graphing the data set
Graph your data set on the set of axis below. Examine your graph. What type of regression does it model? Give several reasons for your conclusion.

The regression equation
Using the regression instructions, find a regression equation that fits your data set. [Note: when finding your regression equation, some calculators won’t allow you to use the initial point where $t = 0$ and $N(0) = 144$. If you do use this you will notice that your calculator will give you a domain error message.] Use one or more of your data points to check your equation. Sketch your regression equation on your data points above.
The regression equation using base $e$
In the last activity, we talked briefly about a base $e$ equation that can also model your function. Let’s write an equation with base $e$ that models this equation. The standard form for an exponential decay is $N(t) = N_0e^{kt}$. In order to make a model, we need to find values for $N_0$ and $k$. Use your data set to find a model with base $e$ for this situation. (Note: you will be using your algebra skills—not your calculator’s graphing function.)

Connection between the equations and the value of $b$ in the regression equation without base $e$ and the base $e$ equation
Compare your two equations—the base $e$ and the base $b$ equation. Discuss the similarities and the differences between the two.

Graphical connection of a radioactive decay model
Compare the graphs of an exponential growth model like we did in the last activity to an exponential decay model as we did in this activity. What are the similarities? What are the differences? You should discuss transformations in your discussion. How can you transform the exponential growth into an exponential decay by changing the equation? [Hint: think about the exponent.]

Now, look at both of the equations that you found for the decay model in this activity. Compare both of them and discuss how both are transformed graphs of an exponential growth. Also discuss the meaning of the exponential growth model and the exponential decay model. What is happening scientifically?

Making Predictions Using the Equations that we found
1. Do you think that one of the models (either the base $b$ or the base $e$) is more accurate than the other? Explain.
2. How much of the radium-222 will remain after 4 minutes? (Pay close attention to your units.)
3. How much of the radium-222 will remain after 2 hours?
4. When will there only be 0.01 grams of the radium left?
5. When will there be 57 grams left?
6. When will there be 120 grams of the substance?

7. When will there be -10 grams of the substance?

8. When will there be 35% of the original amount?

Other Exponential Decay Questions
1. A radioactive substance has been measured periodically in hopes of finding the half-life of the substance. The following table shows the amount left after a certain amount of time.

<table>
<thead>
<tr>
<th>Amount N(t) in mg</th>
<th>219.2 mg</th>
<th>197.6 mg</th>
<th>164.8 mg</th>
<th>120.6 mg</th>
<th>111.6 mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (t) in days</td>
<td>1 day</td>
<td>5 days</td>
<td>12 days</td>
<td>24 days</td>
<td>27 days</td>
</tr>
</tbody>
</table>

First, sketch the graph. Is this an exponential growth or decay model? How do you know?

Find a regression equation for this data using your calculator. Sketch this regression equation on the graph.

Find an equation algebraically for the data using base e.
What is the half-life of this substance? First predict the half-life from your graph. Then predict the half-life from your equations.

When will there be 15 milligrams of the substance left?

How much will remain after 40 days?

When will there only be 15% of the original amount?

2. Carbon 14 has a half-life of 5730 years. If you start with a 500 mg sample, what is a model that will tell you the amount remaining \(N(t)\) after time \(t\)?

What percentage of the carbon will remain after 3000 years?

How much will remain after 2400 years?

When will there be 25 mg remaining?

3. You are examining an unknown substance. You graph the data in terms of how much is remaining after \(t\) days. Your findings are listed on the graph below. What is a model that fits your data?

<table>
<thead>
<tr>
<th>Time in days</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount remaining in grams</td>
<td>10</td>
<td>8.75</td>
<td>7.5</td>
<td>6.5</td>
<td>5.75</td>
<td>4.9</td>
<td>4.2</td>
</tr>
</tbody>
</table>
Connections
Relating previous skills to a new skill; we plan to provide at least one problem on each test which expands previously learned skills to a new application. In this section we will be solving exponential equations both graphically and algebraically; as well as look at exponential functions and their inverses.

Discuss two ways that you can solve the equation $e^{9x+5} = 14$ graphically. Solve the equation in both graphical ways. When you solve them graphically, show where the solution is located on the graph. Now, solve the equation $e^{9x+5} = 14$ algebraically. Compare your graphical solutions to your algebraic solution? Which one is more accurate? Explain.

Sketch the graph of the function $f(x) = e^{x+1} + 5$. Sketch the graph of $f^{-1}(x)$. (Remember that the graph of a function and its inverse are reflections over the $y = x$ line.) Write an equation that you think represents $f^{-1}(x)$. (It is important that you sketch the graph accurately and on graph paper if possible. This will allow for a more accurate solution.)

Now, algebraically find the equation for $f^{-1}(x)$. Compare your graphical and your algebraic solution. Discuss any similarities and any differences.

Exercises
Find the equation for the inverse function both graphically by sketching the function and its inverse and algebraically.

1. $f(x) = e^{x-3} + 2$
2. $g(t) = 4^{t+3} - 2$
3. $h(t) = 3^{2x+5}$
4. $y = \log(x-2) + 1$
5. $k(n) = \log_2(n+3) + 5$
6. $r(x) = \log_3(2x+1) - 6$