Quadratic Equations
Group Activity 3
STEM Project Week #5

In this activity we are going to further explore quadratic equations. We are going to analyze different parts of the quadratic equation (vertex and x-intercepts), solve quadratic equations, and solve quadratic inequalities.

Let's begin by creating another quadratic regression. We will look at the same type of problem that we looked at in Activity 2.

Getting Started with a Data Set

A ball is thrown straight up in the air. The table shows the height, $h$, in feet of the ball after $t$ seconds.

<table>
<thead>
<tr>
<th>time ($t$) in seconds</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>height ($h$) feet</td>
<td>5</td>
<td>89</td>
<td>141</td>
<td>161</td>
<td>149</td>
<td>105</td>
<td>29</td>
</tr>
</tbody>
</table>

First, find a window for your graph. If you need help, refer to the “steps for finding the window” in the regressions document. We have used these steps in the last two activities. Once you have found the window, label it on the grid below. Now plot your scatterplot. (Remember the vertical axis is $h$ (height) and the horizontal axis is $t$ (time).)

If you plotted the points correctly you should see that a quadratic fits the data best. Use the directions in the Regression Instructions document to find and graph the quadratic regression on your calculator.

What is the quadratic equation that best fits this data? (write your answer using \( t \) and \( h \))

What is the window that you have used for your calculator?

What is the domain of your function? (Think about what the domain represents.)

What is the range of your function? (Think about what the range represents.)

Sketch your regression graph on this paper just as it appears on your calculator. Sketch it below labeling the window.
Now we will examine different parts of your parabola.

**The Vertex of a Parabola**

In this section we will examine the vertex of the parabola. We will find the vertex on the graph. We will find the vertex algebraically and we will determine what it means in our context.

First, locate and label the vertex on the graph that you sketched. What are the approximate coordinates of the vertex?

Now, let's find the vertex on your calculator using the maximum key. Notice that the vertex of a quadratic is either the highest point (largest \( h \)-value) or lowest point (smallest \( h \)-value) on your graph. Thus, if your parabola opens down it is your largest \( h \)-value or it is a *maximum* value. If your parabola opens up the vertex is the smallest \( h \)-value or it is a *minimum* value. (In math we would write the \( y \)-value.)

Looking at your graph does it have a *maximum* or a *minimum* value?

Using the points that you approximated for the vertex, what is the *maximum* or *minimum* value? (The *value* is the \( h \)-coordinate.)

Now, we will find the *maximum/minimum* value using your calculator. Once you have the graph on your screen use the following steps:

**Find the Maximum/Minimum Value using TI-83**

1. Press Calc (2nd Trace)
2. Choose Maximum (#4) [if your parabola opens up you will choose Minimum #3]
3. For the Left Bound, arrow to the left of the vertex and press enter.
4. For the Right Bound, arrow to the right of the vertex and press enter.
5. For the Guess, arrow to the vertex and press enter.

The TI-83 will return a value for \( x \) and \( y \). The \( x \) is where the vertex occurs and the \( y \) is the maximum or minimum value. (Remember we need to use \( x \) and \( y \) for the calculator, but we are working in \( t \) and \( h \), respectively, on paper.)

What is your maximum value and what \( t \)-value is it located at?

Is your sketched vertex close to your calculated vertex? If it is not, examine why.
What does this value mean in our context? (Remember that the $t$-value is your time and the $h$-value is the height of the ball.)

In your last answer did you indicate your unit value with s and ft? If not, please go back and do this.

**Finding the Vertex Algebraically**

Now let's find the vertex algebraically. The vertex is located at the coordinates: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$. Use the regression equation that you found at the beginning of this activity to find the vertex using our above formulas.

Is the vertex that you found algebraically close to the vertex that you found using your calculator? If not, explain why.

**X-intercepts of Your Graph and Solving a Quadratic Equation**

We have three ways that we can find the $t$-intercepts of the graph of the quadratic equation. We can locate them on the graph, we can find them algebraically, and we can find them using our calculator. In this section, we will find them using all three of these methods.

On your sketched graph, locate and label the $t$-intercepts.

What do your $t$-intercepts represent in the context of this problem?

Two important questions:

Question 1) How are the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and the vertex directly related? In other words what is the relationship between the $x$-intercepts, the line of symmetry, the vertex and the quadratic formula?

Question 2) What does $f\left(\frac{-b}{2a}\right)$ mean?

If your instructor has not already explained, please feel free to ask them explain this to the entire group at this time.
Next, we will find the t-intercepts algebraically. The t-intercepts occur when the height of the ball is _______________. (Did you use units?)

Therefore, the equation that we solve to find the t-intercepts is ______________________.

We now have a quadratic equation. We have three ways that we can algebraically solve a quadratic equation. We can set the equation equal to zero and factor, we can set the equation equal to zero and use the quadratic formula, and we can set the equation equal to the constant and complete the square. Solve the equation. What are the t-intercepts of your quadratic equation?

Another way to find these t-intercepts is to use your calculator. (Switched to x because of calculator limitations)

Once you have the quadratic equation graphed on your calculator you may find the intercepts using the CALC button (2nd Trace). Find the first t-intercept.

Repeat this to find second t-intercept.

What are the t-intercepts that you found using your calculator? (What does the left most t-intercept represent?)

Here is a second data set to practice with

<table>
<thead>
<tr>
<th>time (t) in seconds</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>height (h) feet</td>
<td>100</td>
<td>149</td>
<td>200</td>
<td>242</td>
<td>268</td>
<td>285</td>
<td>295</td>
</tr>
</tbody>
</table>

Graph and find the maximum height, the time of the maximum height and the time when the ball hits the ground.
Solving a Quadratic Inequality

Relating previously learned skills to a new skill: We plan to provide at least one problem on each test which expands previously learned skills to a new application. Today we will expand on solving inequalities as an example of how this works.

Two problems that are related to previous work are:
- Graph the quadratic inequality \( y > 2x^2 - 5x - 1 \)
- Solve the quadratic inequality \( 0 > 2x^2 - 5x - 1 \)

These are related to three previous problems:
- Graph on the number line \( x = 3 \)
- Graph on the number line \( x > 3 \)
- Graph \( y = 2x^2 - 5x - 1 \)

To begin let’s graph \( x = 3 \) and \( x > 3 \)

\[
\begin{align*}
\text{Graph } & x = 3 \\
\text{Graph } & x > 3
\end{align*}
\]

In words, explain the differences between the graph of the equality and the graph of the inequality. Include the changes that were made at 3 and what the arrow represents and why.

Next graph \( y = 2x^2 - 5x - 1 \)
Using the same methods that you changed \( x = 3 \) into \( x > 3 \), change the above quadratic graph to represent \( y > 2x^2 - 5x - 1 \). Note: On the number line, you change the endpoint into an open circle when the inequality does not include an equals sign. This is not possible when graphing a quadratic. The subtle change is to use a dotted or dashed line to represent the fact that the quadratic is greater than but not equal to. Also, you need to shade the entire side of the quadratic that is true.

Finally, to solve \( 0 > 2x^2 - 5x - 1 \), notice that you have changed the \( y \) to a zero and \( y = 0 \) is the \( x \)-axis. Thus the solution is represented by your graph above the \( x \)-axis. Before asking for help, ponder this for a moment.

Some problems to practice:

1. Graph \( y \leq 3x^2 - 4x - 5 \) and solve \( 0 \leq 3x^2 - 4x - 5 \)
2. Graph \( y > -2.3x^2 - 4.7x + 3 \) and solve \( 0 > -2.3x^2 - 4.7x + 3 \)
3. Graph \( y < -2.7x^2 - 4.3x + 1.1 \) and solve \( 0 \leq -2.7x^2 - 4.3x + 1.1 \)

Question to ponder.

When solving the inequality \( 0 > 2x^2 - 5x - 1 \), we graphed \( y > 2x^2 - 5x - 1 \) and looked at the \( x \)-axis; since the \( x \)-axis is the line \( y = 0 \). Using this information, determine two methods to solve the inequality \( 5 > 2x^2 - 5x - 1 \).