A Proof that $X^3 + Y^3 = Z^3$ has no solutions in positive integers.

Andrew Wiles’ proof of Fermat’s Last Theorem, that $X^n + Y^n = Z^n$ ($n \geq 3$) has no solutions in positive integers, is famously difficult. Although Kummer fell short of a proof in the most general case, he was able to prove that for a prime $p \geq 3$, $X^p + Y^p = Z^p$ has no positive integer solutions assuming $p$ does not divide the class number of $\mathbb{Q}(\zeta_p)$, where $\zeta_p$ is the $p$th root of unity. Using algebraic properties of $\mathbb{Z}[\zeta_3] = \{a+b\zeta_3 : a, b \in \mathbb{Z}\}$ ($\zeta_3 = \frac{-1 + \sqrt{-3}}{2}$), we will prove that $X^3 + Y^3 = Z^3$ has no solutions in positive integers. Time permitting, we will prove that the class number of $\mathbb{Q}(\zeta_3)$ is 1 and discuss the relationship between the class number and unique factorization.