Quadratic Equations
Group Activity 3
Business Project Week #5

In this activity we are going to further explore quadratic equations. We are going to analyze different parts of the quadratic equation (vertex and x-intercepts), solve quadratic equations, and solve quadratic inequalities.

Let’s begin by creating both another linear and another quadratic regression.

**Getting Started with a Data Set**

A cell phone manufacturing company used market research to determine the demand for cell phones based on price.

<table>
<thead>
<tr>
<th>Price (p)</th>
<th>$0</th>
<th>$10</th>
<th>$30</th>
<th>$60</th>
<th>$100</th>
<th>$150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (n_d)</td>
<td>35000</td>
<td>32900</td>
<td>29400</td>
<td>22500</td>
<td>14900</td>
<td>4600</td>
</tr>
</tbody>
</table>

While not a perfect line. The market research has determined that the number of cell phones in demand is linear. Use the calculator to determine a model (mathematical equation) to approximate the number of cell phones in demand based on price.

What is the equation?

The same market research project also determined the number of cell phones that manufactures would supply based on price. The next set of data gives the number of cell phones that would be manufactured at a given price.

<table>
<thead>
<tr>
<th>Price (p)</th>
<th>$10</th>
<th>$30</th>
<th>$60</th>
<th>$100</th>
<th>$140</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (n_s)</td>
<td>5000</td>
<td>18000</td>
<td>30000</td>
<td>22000</td>
<td>2000</td>
</tr>
</tbody>
</table>

This data is not linear. At first, as the price goes up, the supply increases as there is more and more profit to be made. But after a while, the number of cell phones that are demanded drops to a point where some manufactures will determine that they would be better off moving out of the

[In this example, we are letting n represent the number of cell phones. In the first graph, we are interested in the number of cell phones based on demand. And in the second graph, we are interested in the number of cell phones that will be supplied. Since we are interested in the number of cell phones, both will be denoted with the letter n. And both will be graphed on the vertical axis. But to keep the demand and the supply values separate, we will denote the number of cell phones in demand by n_d and the number related to supply as n_s.]
competition and start manufacturing something else. Hence the number of cell phones manufactured will begin to drop off.

The market research indicates that the best curve for the supply data is a quadratic equation. Use a quadratic regression to determine the supply equation for the second set of data given.

What is the supply equation, round coefficients to the nearest thousandths?

Based on the supply equation, how many cell phones would be produced if the price was $3.00?

Does the last answer make sense? Why or why not?

The domain of a quadratic equation is all real numbers. Hence $3.00 is in the domain of the quadratic equation but the answer is meaningless in the context of the discussion on cell phones. In mathematics, when we build models such as the quadratic model for supply, we restrict the domain of the problem to where the equation makes sense. Take a moment to describe how you would restrict the domain in this problem: Write down your thoughts:

When we restrict the domain, we call the restricted domain the implied domain as it is the domain implied by the conditions the equation was created for. In this case, we know that it is silly to discuss the manufacturing of a negative number of cell phones. Thus the implied domain is restricted only for values where the supply is positive or zero. How can we find the implied domain?
The \( p \)-intercepts (note: this really is the \( x \)-intercepts since price is on the horizontal axis where the \( x \) values typically go), indicate the points where the supply moves from positive to negative. Thus, the \( p \)-intercepts actually define the boundaries of the implied domain. Find the \( p \)-intercepts and state the implied domain.

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**\( x \)-intercepts of Your Graph and Solving a Quadratic Equation**

We will quickly discuss how to use your calculator to find the \( p \)-intercepts. We will switch to calling this the \( x \)-intercepts and using \( x \) and \( y \) to represent the values of \( p \) and \( n \). The change back to \( x \) and \( y \) is caused by limitations of the calculator. The ability to think of a problem in the terms of the letters given and to think about the letters as \( x \) and \( y \) is an important calculator skill.

Enter the quadratic equation into your calculator, change the window to appropriate settings (what are these?) and graph the equation.

Once graphed, press the second key (blue key) and then press the trace button (second from the right on the very top). Select zero, this is \#2 in the list, and you be taken to the graph where you will have to move the cursor to the left of the zero you are looking for and press enter. Next move the cursor to the right of the desired zero and press enter. Move the cursor close to the vertex for a guess and press enter a third time, the answer will appear at the bottom of the graph.

Write down your first zero.

Repeat for second zero.

What is the implied domain?
**Equilibrium points**

In business, equilibrium of supply and demand occurs when \( n_d = n_s \). This is called equilibrium because at this price, we are manufacturing exactly the number of cell phones desired. If we change the price slightly, we will:

- Manufacture too many cell phones, which means the warehouses will fill up and the price will be lowered with sales.
- Manufacture too few, which means the demand will create a need for more cell phones and allow a window for competitors to start manufacturing our product.

To determine the equilibrium points for this problem, press the Y= button and enter the linear demand equation originally created into \( y_2 \). Press graph. With the quadratic demand equation already in \( y_1 \), both equations should be graphed at this time.

The two equations should have two visible points of intersection (these are the only two points). Thus, there are two points of equilibrium. Press the second key followed by the trace button to find the word intersect. Arrow down to intersect. Press enter. \( Y_1 \) should show up at the top of the screen, press enter. \( Y_2 \) now shows up at the top of the screen, press enter. Move the cursor close to one of the points of intersection and press enter. This will give you the first equilibrium point.

Write down the equilibrium point in \( (p, n) \) form.

Repeat and find the second equilibrium point.

Two equilibrium points: Think about this, what does this mean?

Want an answer to the last question, stay engaged in your business class and ask when the opportunity arises.

**The Vertex of a Parabola**

While the vertex of the parabola is not important in this context, it is worth calculating. Use the second and trace buttons to find maximum and calculate the vertex. In this problem, the vertex is a maximum, but be careful as in other quadratic problems the vertex is a minimum.

What is the vertex?
Solving a Quadratic Inequality

Relating previously learned skills to a new skill: We plan to provide at least one problem on each test which expands previously learned skills to a new application. Today we will expand on solving inequalities as an example of how this works.

Two problems that are related to previous work are:
- Graph the quadratic inequality \( y > 2x^2 - 5x - 1 \)
- Solve the quadratic inequality \( 0 > 2x^2 - 5x - 1 \)

These are related to three previous problems:
- Graph on the number line \( x = 3 \)
- Graph on the number line \( x > 3 \)
- Graph \( y = 2x^2 - 5x - 1 \)

To begin let’s graph \( x = 3 \) and \( x > 3 \)

\[
\begin{align*}
x &= 3 \\
0 &\quad 0
\end{align*}
\]

In words, explain the differences between the graph of the equality and the graph of the inequality. Include the changes that were made at 3 and what the arrow represents and why.

Next graph \( y = 2x^2 - 5x - 1 \)
Using the same methods that you changed \( x = 3 \) into \( x > 3 \), change the above quadratic graph to represent \( y > 2x^2 - 5x - 1 \). Note: On the number line, you change the endpoint into an open circle when the inequality does not include an equals sign. This is not possible when graphing a quadratic. The subtle change is to use a dotted or dashed line to represent the fact that the quadratic is greater than but not equal to. Also, you need to shade the entire side of the quadratic that is true.

Finally, to solve \( 0 > 2x^2 - 5x - 1 \), notice that you have changed the \( y \) to a zero and \( y = 0 \) is the \( x \)-axis. Thus the solution is represented by your graph above the \( x \)-axis. Before asking for help, ponder this for a moment.

Some problems to practice:

1. Graph \( y \leq 3x^2 - 4x - 5 \) and solve \( 0 \leq 3x^2 - 4x - 5 \)
2. Graph \( y > -2.3x^2 - 4.7x + 3 \) and solve \( 0 > -2.3x^2 - 4.7x + 3 \)
3. Graph \( y < -2.7x^2 - 4.3x + 1.1 \) and solve \( 0 \leq -2.7x^2 - 4.3x + 1.1 \)

Question to ponder.

When solving the inequality \( 0 > 2x^2 - 5x - 1 \), we graphed \( y > 2x^2 - 5x - 1 \) and looked at the \( x \)-axis; since the \( x \)-axis is the line \( y = 0 \). Using this information, determine two methods to solve the inequality \( 5 > 2x^2 - 5x - 1 \).