Waring rank is a measure of complexity of polynomials related to sums-of-powers expressions and to a number of applications such as interpolation problems, blind source separation problems in signal processing, mixture models in statistics, and more. We review some recent advances having in common the use of apolarity, a sort of reversed version of differential equations in which one considers the set of differential equations that have a given function as a solution.

1. Apolarity is applied to describe criteria for a polynomial to be expressible as a sum of functions in separate sets of variables, possibly after a change of coordinates. This is related to separation-of-variables techniques in differential equations and to topology (criteria for a manifold to be decomposable as a connected sum). This is joint work with Buczynska, Buczynski, and Kleppe.

2. The set of sum-of-powers decompositions of a monomial is described. A corollary is a necessary and sufficient condition for a monomial to have a unique such decomposition, up to scaling the variables. This is joint work with Buczynska and Buczynski.

3. One generalization of monomials is the family of polynomials that completely factor as products of linear factors, geometrically defining a union of hyperplanes. Waring ranks of hyperplane arrangements are determined in the case of mirror arrangements of finite reflection groups satisfying a technical hypothesis which includes many cases of interest. This is joint work with Woo.

If time permits, ongoing work will be described, including geometric lower bounds for generalized Waring rank, apolarity of general hyperplane arrangements, and a number of other open questions.