$\mathcal{SPM}$ BULLETIN

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1. Editor's note

Never has an issue of the $\mathcal{SPM}$ **Bulletin** contained as much interesting information as this issue does. In addition to the interesting research announcements, this issue contains announcements of solutions for *three* open problems, one of which being a *Problem of the month* in an earlier issue.

Contributions to the next issue are, as always, welcome.

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2. Research announcements

2.1. **The Reznichenko property and the Pytkeev property in hyperspaces.** We investigate two closure-type properties, the Reznichenko property and the Pytkeev property, in hyperspace topologies.

`http://arxiv.org/abs/math.GN/0312477`
Ljubisa D. R. Kocinac
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2.2. **Between Maharam's and von Neumann's problems.** If I is a suitably definable σ-ideal on the real line and the factor algebra of Borel sets modulo I is weakly distributive then the algebra carries a Maharam submeasure.

`http://arxiv.org/abs/math.LO/0401134`
Ilijas Farah and Jindrich Zapletal
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2.3. **Fraisse Limits, Ramsey Theory, and Topological Dynamics of Automorphism Groups.** We study in this paper some connections between the Fraisse theory of amalgamation classes and ultrahomogeneous structures, Ramsey theory, and topological dynamics of automorphism groups of countable structures.

`http://arxiv.org/abs/math.LO/0305241`
A. S. Kechris, V. G. Pestov and S. Todorcevic

2.4. **Concerning problems about cardinal invariants on Boolean algebras.** The present status of the problems in my book *Cardinal Invariants on Boolean algebras* (Birkhauser 1996) is described, with a description of solutions or partial solutions, and references.

`http://arxiv.org/abs/math.LO/0401343`
James Donald Monk
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2.5. **Small Locally Compact Linearly Lindelof Spaces.** There is a locally compact Hausdorff space of weight $\aleph_\omega$ which is linearly Lindelof and not Lindelof. This improves an earlier result, which produced such a space of weight $\beth_\omega$.

`http://arxiv.org/abs/math.GN/0402066`
Kenneth Kunen

2.6. **Luzin gaps.** We isolate a class of $F_{\sigma\delta}$ ideals on $\mathbb{N}$ that includes all analytic P-ideals and all F_σ ideals, and introduce 'Luzin gaps' in their quotients. A dichotomy for Luzin gaps allows us to freeze gaps, and prove some gap preservation results. Most importantly, under PFA all isomorphisms between quotient algebras over these ideals have continuous liftings. This gives a partial confirmation to the author's rigidity conjecture for quotients $\mathcal{P}(\mathbb{N})/\mathcal{I}$. We also prove that the ideals NWD($\mathbb{Q}$) and NULL($\mathbb{Q}$) have the Radon–Nikodým property, and (using OCA_∞) a uniformization result for $\mathcal{K}$-coherent families of continuous partial functions.

To appear in: *Transactions of the AMS.*

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2.7. **The weak Fréchet-Urysohn property in function spaces.** For a Tychonoff space X, we denote by $C_p(X)$ the space of all real-valued continuous functions on X with the topology of pointwise convergence. In this paper, we note that for every analytic space X, $C_p(X)$ is weakly Fréchet-Urysohn, and solve a related problem of Tsaban [6].

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The solved problem. Following [2], we say that a cover $\mathcal{U}$ of X is $\mathfrak{U}$-*groupable* if there is a partition of $\mathcal{U}$ into finite sets, $\mathcal{U} = \bigcup_{n\in\mathbb{N}} \mathcal{F}_n$, such that for each infinite subset A of $\mathbb{N}$, $\{\cup\mathcal{F}_n\}_{n\in A} \in \mathfrak{U}$. Let $\mathfrak{U}^{gp}$ be the family of $\mathfrak{U}$-groupable elements of $\mathfrak{U}$.

In [2] it is proved that the Hurewicz covering property $\mathsf{U}_{fin}(\mathcal{O},\Gamma)$ is equivalent to $\mathsf{S}_{fin}(\Lambda,\Lambda^{gp})$. In [7] it is proved that $\mathsf{S}_{fin}(\Lambda,\Lambda^{gp}) = \binom{\Lambda}{\Lambda^{gp}}$. In [6] we asked whether the analogue result for $\binom{\Omega}{\Omega^{gp}}$ is true, namely, whether $\mathsf{S}_{fin}(\Omega,\Omega^{gp}) = \binom{\Omega}{\Omega^{gp}}$.

Sakai [§2.7 above] gave a negative answer in the following strong sense: He showed that the Baire space ${}^{\mathbb{N}}\mathbb{N}$ (and, consequently, any analytic space) satisfies the stronger property $\binom{\mathcal{B}_\Omega}{\mathcal{B}_\Omega^{gp}}$. Recall that the Baire space does not even satisfy $\mathsf{S}_{fin}(\mathcal{O},\mathcal{O})$ (Menger's property), which is the weakest property in the Scheepers diagram.

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2.8. **$\mathcal{F}$-Hurewicz spaces.** We investigate a generalization of spaces that satisfy the Hurewicz covering property. In particular we are interested in characterization of such spaces in terms of some properties of function spaces.

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2.9. **Open problems in topology.** A new survey of the book *Open Problems in Topology* appeared in *Topology and its Applications* **136** (2004) 37–85. The survey and the original book are available from the Elsevier Mathematics portal

`http://www.elseviermathematics.com/`

(select *Books* and go to the bottom of the page).

I have also completed editing *Problems from Topology Proceedings.* The book is available online at

`http://at.yorku.ca/i/a/a/z/10.htm`
`http://arXiv.org/abs/math.GN/0312456`

This book consists of material originally appearing in the Problem Section of the journal *Topology Proceedings* since 1976 as well as some other well-known problem lists in general topology from the 1970's that have some connection to the journal. The problems have been updated with current information on solutions with bibliographic references. In particular, the book features these collections:

- All contributed problems to the Problem Section, classified by subject;
- Eight Classic Problems by Peter J. Nyikos, including information from the two recent articles Twenty-five years later;
- New Classic Problems from 1990;
- Problems from Mary Ellen Rudin's Lecture notes in set-theoretic topology (1975/7);
- Problems from A.V. Arhangelskii's Structure and classification of topological spaces and cardinal invariants (1978);
- Continuum theory problems by Wayne Lewis (1983);
- Problems in continuum theory by Janusz R. Prajs, including essays by Charles L. Hagopian and Janusz J. Charatonik;
- Classification of homogeneous continua by James T. Rogers, Jr., including material from survey articles of 1983 and 1989.

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2.10. **The hyper-weak distributive law and a related game in Boolean algebras.** The hyper-weak (κ, λ)-distributive law, formulated by Prikry, is a very weak generalization of the $(\kappa, \lambda, < \mu)$-distributive law. We define a related infinitary, two-player game, called $\mathcal{G}^{\kappa}_{\lambda-1}$, and give connections between the hyper-weak (κ, λ)-distributive law and the existence of winning strategies for the two players of $\mathcal{G}^{\kappa}_{\lambda-1}$, obtaining a game-theoretic characterization of the hyper-weak (κ, λ)-distributive law for many pairs of cardinals κ, λ, under GCH. We then construct κ^+-Suslin algebras for every infinite cardinal κ in which, for each infinite cardinal $\lambda \leq \kappa$, neither player has a winning strategy for $\mathcal{G}^{\kappa}_{\lambda-1}$. This shows that the gap between the strengths of the properties "II wins $\mathcal{G}^{\kappa}_1(\infty)$ in $\mathbb{B}$" and "the (κ, ∞)-distributive law holds in $\mathbb{B}$" is consistently even larger than was previously known.

For a related work see: Natasha Dobrinen, *Games and general distributive laws in Boolean algebras*, Proc. AMS **131** (2003), 309–318. (See also: Natasha Dobrinen, *Errata to 'Games and general distributive laws in Boolean algebras'*, Proc. AMS **131** (2003), 2967–2968.)

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2.11. **Problem of Issue 7 solved.** In the seventh issue we have posed the following conjecture.

Conjecture 2.1. *If X has strong measure zero and $|X| < \mathfrak{b}$, then all finite powers of X have strong measure zero.*

Bartoszynski has found a combinatorial proof of the conjecture when X is a subsets of the Cantor space (to appear in [8]). We have very recently found out that Scheepers has proved the following more general result in [5], for arbitrary metric spaces: If X has the Hurewicz property $\mathsf{U}_{fin}(\mathcal{O}, \Gamma)$ and strong measure zero, and Y has strong measure zero, then $X \times Y$ has strong measure zero. A similar result follows from [3], as shown in [8].

The following, though, remains open.

Problem 2.2. *Assume that X satisfies the Hurewicz property $\mathsf{U}_{fin}(\mathcal{O}, \Gamma)$ and has strong measure zero (the last property can be replaced by $\mathsf{S}_1(\mathcal{O}, \mathcal{O})$ or meager-additive). Does it follow that all finite powers of X satisfy $\mathsf{U}_{fin}(\mathcal{O}, \Gamma)$?*

A positive answer would imply that the Gerlitz-Nagy property $(*)_{GN} = \mathsf{S}_1(\Omega, \Lambda^{gp})$ is preserved under taking finite powers, and that $(*)_{GN} = \mathsf{S}_1(\Omega, \Omega^{gp})$, see the definitions in [§2.7 above].

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2.12. **Errata to: *Hereditary topological diagonalizations and the Menger-Hurewicz Conjectures*.** Taras Banakh and Lubomyr Zdomsky have found a gap in our mentioned paper. A revision of the paper has been made, and the results promised in the abstract of the original paper are still proved. Currently the paper is being thoroughly checked by a colleague. Once verified for correctness, it will be re-posted to the Mathematics ArXiv.

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2.13. **The additivity number of the Menger and Scheepers properties.** For a family $\mathcal{I}$ of subspaces of a topological space, define $\mathsf{add}(\mathcal{I}) = \min\{|\mathcal{F}| : \mathcal{F} \subseteq \mathcal{I}$ and $\cup\mathcal{F} \notin \mathcal{I}\}$. I have recently obtained the following results. For subspaces of a hereditarily Lindelöf topological space:

(1) $\mathsf{add}(\mathsf{U}_{fin}(\Gamma, \mathcal{O})) \geq \mathfrak{g}$,
(2) Under NCF, $\mathsf{add}(\mathsf{U}_{fin}(\Gamma, \Omega)) = \mathfrak{d}$.

The first result gives a negative answer to Problem 2.4 from [1, full version], which asks whether $\mathsf{add}(\mathsf{U}_{fin}(\Gamma, \mathcal{O})) = \mathfrak{b}$. Indeed, under $\mathfrak{u} < \mathfrak{g}$ we have $\mathfrak{b} < \mathfrak{g}$.

The second result is a straightforward translation of some other results of Taras Banakh, and it strengthens the result of [1, full version], that under NCF, $\mathsf{add}(\mathsf{U}_{fin}(\Gamma, \Omega)) \geq \max\{\mathfrak{b}, \mathfrak{g}\}$.

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3. Conferences

3.1. **Geometric Topology: Infinite-Dimensional Topology, Absolute Extensors, Applications.** The conference will be held on 26–29 of May 2004, at the Lviv Ivan Franko National University, Lviv, Ukraine.

Organizing/Program Committee: T. Banakh, R. Cauty, A. Chigogidze, J.E. Keesling, V. Kyrylych, Ya. Prytula, D. Repovs, O. Skaskiv, M. Zarichnyi.

We plan plenary talks (45 min), section talks (25 min) and short communications (10 min). The Conference will be held in the Main Building of the University in the historical center of Lviv. The official language of the Conference is English. Persons interested in participating at the Conference are kindly asked to register by e-mail as well as to send one page abstract in English, which should be prepared in LaTeX by e-mail till March 30, 2004. The registration fee is 50 USD (for accompanying persons 25 USD), for participants from the countries of the former Soviet Union the registration fee is 20 USD (for accompanying persons 10 D). The registration fee is to be paid upon arrival in Lviv. The fee covers organization costs: abstracts, tea/coffee during breaks, and cultural program.

The accomodation price in Lviv at the moment is from 10 to 80 USD per night depending on facilities. For further information, please visit the following web-sites:

`http://www.all-hotels.com.ua/addz2.php3?Lang=1&sCity=4`
`http://www.piligrim.lviv.ua/ukraine/page6_en.html`

The Organizing Committee will have an opportunity to pre-book rooms in Students Hostel (for interested participants). We regret that travel and daily expenses cannot be paid by the Organizing Committee.

For further information or specific requests please e-mail us to the following address.

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4. Problem of the month

For an ideal $\mathcal{J} \subseteq P(\mathbb{R})$, we say that $H \subseteq \mathbb{R} \times \mathbb{R}$ is a *Borel $\mathcal{J}$-set* if $(H)_x \in \mathcal{J}$ for all $x \in \mathbb{R}$.

Using this terminology we define the following classes of small sets:

$$\begin{aligned}
\mathsf{COF}(\mathcal{J}) &= \{X \subseteq \mathbb{R} : \text{for every Borel } \mathcal{J}\text{-set } H, \{(H)_x : x \in X\} \text{ is not a basis of } \mathcal{J}\},\\
\mathsf{ADD}(\mathcal{J}) &= \{X \subseteq \mathbb{R} : \text{for every Borel } \mathcal{J}\text{-set } H, \textstyle\bigcup_{x\in X}(H)_x \in \mathcal{J}\},\\
\mathsf{COV}(\mathcal{J}) &= \{X \subseteq \mathbb{R} : \text{for every Borel } \mathcal{J}\text{-set } H, \textstyle\bigcup_{x\in X}(H)_x \neq \mathbb{R}\},\\
\mathsf{NON}(\mathcal{J}) &= \{X \subseteq \mathbb{R} : \text{every image of } X \text{ by a Borel function is in } \mathcal{J}\}.\\
\mathsf{D} &= \{X \subseteq \mathbb{R} : \text{ for every Borel function } f : \mathbb{R} \to {}^{\mathbb{N}}\mathbb{N},\ f[X] \text{ is not dominating}\}.\\
\mathsf{B} &= \{X \subseteq \mathbb{R} : \text{ for every Borel function } f : \mathbb{R} \to {}^{\mathbb{N}}\mathbb{N},\ f[X] \text{ is bounded}\}.
\end{aligned}$$

The interrelationships of these classes were extensively studied in [4] in the case that $\mathcal{J}$ is $\mathcal{M}$ (the ideal of meager sets of reals) or $\mathcal{N}$ (the ideal of Lebesgue measure zero sets of reals). These are summarized in Figure 1.

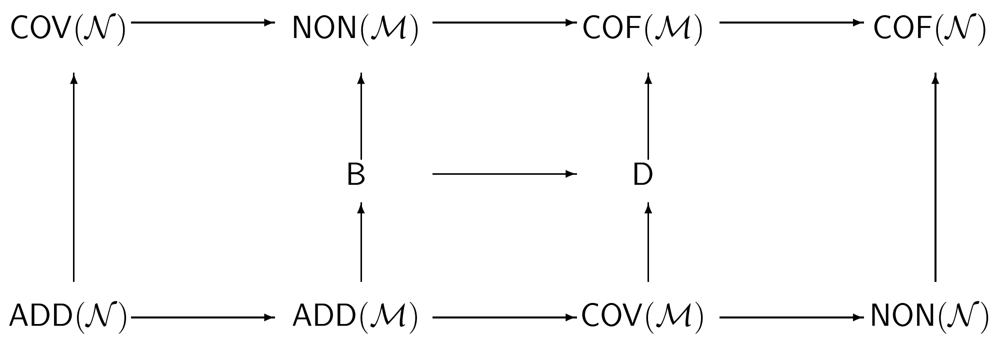

FIGURE 1. Chichon Diagram for small sets of reals

The relationship between Figure 1 and the well-known Chichon diagram that expresses provable relationships among certain cardinal numbers is that a cardinal number in a particular position in Cichon's diagram is actually the minimal cardinality for a set of real numbers not belonging to the class in the corresponding position in Figure 1.

Many of these classes can be defined by selection principles, e.g.: $\mathsf{D} = \mathsf{S}_1(\mathcal{B}_\Gamma, \mathcal{B})$, $\mathsf{B} = \mathsf{S}_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)$, $\mathsf{COV}(\mathcal{M}) = \mathsf{S}_1(\mathcal{B}, \mathcal{B})$, $\mathsf{ADD}(\mathcal{M}) = \mathsf{S}_1(\mathcal{B}, \mathcal{B}) \cap \mathsf{S}_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)$.

The following problem was suggested by Bartoszynski: It is known [4] that if $X \notin \mathsf{NON}(\mathcal{M})$ and $Y \notin \mathsf{S}_1(\mathcal{B}_\Gamma, \mathcal{B})$, then $X \times Y \notin \mathsf{COF}(\mathcal{M})$.

Problem 4.1. *Suppose that $X \notin \mathsf{NON}(\mathcal{M})$ and $Y \notin \mathsf{D}$. Does it imply that $X \cup Y \notin \mathsf{COF}(\mathcal{M})$?*

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5. Problems from earlier issues

In this section we list the past problems posed in the $\mathcal{SPM}$ **Bulletin**, in the section *Problem of the month*. For definitions, motivation and related results, consult the corresponding issue.

For conciseness, we make the convention that all spaces in question are zero-dimentional, separable metrizble spaces.

Issue 1. *Is* $\binom{\Omega}{\Gamma} = \binom{\Omega}{\mathrm{T}}$*?*

Issue 2. *Is* $\mathsf{U}_{fin}(\Gamma, \Omega) = \mathsf{S}_{fin}(\Gamma, \Omega)$*? And if not, does* $\mathsf{U}_{fin}(\Gamma, \Gamma)$ *imply* $\mathsf{S}_{fin}(\Gamma, \Omega)$*?*

Issue 3. *Does there exist (in ZFC) a set satisfying* $\mathsf{U}_{fin}(\mathcal{O}, \mathcal{O})$ *but not* $\mathsf{U}_{fin}(\mathcal{O}, \Gamma)$*?*

Solution. **Yes** (Lubomyr Zdomsky, 2003). □

Issue 4. *Does* $\mathsf{S}_1(\Omega, \mathrm{T})$ *imply* $\mathsf{U}_{fin}(\Gamma, \Gamma)$*?*

Issue 5. *Is* $\mathfrak{p} = \mathfrak{p}^*$*?*

Issue 6. *Does there exist (in ZFC) an uncountable set satisfying* $\mathsf{S}_1(\mathcal{B}_\Gamma, \mathcal{B})$*?*

Issue 7. *Assume that* X *has strong measure zero and* $|X| < \mathfrak{b}$*. Must all finite powers of* X *have strong measure zero?*

Solution. **Yes** (Scheepers [5]; Bartoszynski). See [§2.11 above]. □

References

[1] T. Bartoszynski, S. Shelah, and B. Tsaban, *Additivity properties of topological diagonalizations*, The Journal of Symbolic Logic **68** (2003), 1254–1260. (Full version: `http://arxiv.org/abs/math.LO/0112262`)

[2] Lj. D. R. Kočinac and M. Scheepers, *Combinatorics of open covers (VII): groupability*, Fundamenta Mathematicae **179** (2003), 131–155.

[3] A. Nowik, M. Scheepers, and T. Weiss, *The algebraic sum of sets of real numbers with strong measure zero sets*, J. Symbolic Logic **63** (1998), 301–324.

[4] J. Pawlikowski and I. Recław, *Parametrized Cichoń's diagram and small sets*, Fundamenta Mathematicae **147** (1995), 135–155.

[5] M. Scheepers, *Finite powers of strong measure zero sets*, The Journal of Symbolic Logic **64** (1999), 1295–1306.

[6] B. Tsaban, *The minimal cardinality where the Reznichenko property fails*, Israel Journal of Mathematics **140** (2004), 367–374. `http://arxiv.org/abs/math.GN/0304024`

[7] B. Tsaban, *The Hurewicz covering property and slaloms in the Baire space*, submitted. `http://arxiv.org/abs/math.GN/0301085`

[8] B. Tsaban and T. Weiss, *Products of special sets of real numbers*, eprint `http://arxiv.org/abs/math.LO/0307226`

Previous issues. The first issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, on `http://arxiv.org/abs/math.GN/`x, where x is `0301011`, `0302062`, `0303057`, `0304087`, `0305367`, `0312140`, and `0401155`, respectively, for issues number 1 to 7.

Contributions. Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in LaTeX. The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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