

## SPM BULLETIN

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### 1. EDITOR'S NOTE

The second SPM meeting promises to be a most interesting event. It suffices to skim the list of participants. Visit

<http://www.matematica.unile.it/ricerca/CSGT/workshop.htm>

and

<http://diamond.boisestate.edu/spm/Lecce2/index.htm>

for details. Those who have not yet submitted their abstract to this meeting are advised to do so at their earliest convenience, at

<http://atlas-conferences.com/cgi-bin/abstract/submit/caqh-01>

Once the abstracts are submitted, it will be possible to see them at

<http://at.yorku.ca/cgi-bin/amca/caqh-01>

Contributions to the next issue are, as always, welcome.

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## 2. RESEARCH ANNOUNCEMENTS

**2.1. New reals: Can live with them, can live without them.** We give a simple proof of the preservation theorem for proper countable support iterations known as “tools-preservation”, “Case A” or “first preservation theorem” in the literature. We do not assume that the forcings add reals.

Currently the paper contains drafts of the proofs only. We will fill in more details soon.

<http://arxiv.org/abs/math.LO/0505471>

*Martin Goldstern and Jakob Kellner*

**2.2. Uniform almost everywhere domination.** We explore the interaction between Lebesgue measure and dominating functions. We show, via both a priority construction and a forcing construction, that there is a function of incomplete degree that dominates almost all degrees. This answers a question of Dobrinen and Simpson, who showed that such functions are related to the proof-theoretic strength of the regularity of Lebesgue measure for  $G_\delta$  sets. Our constructions essentially settle the reverse mathematical classification of this principle.

<http://arxiv.org/abs/math.LO/0506019>

*Peter Cholak, Joseph Miller, and Noam Greenberg*

**2.3. Heredity of  $\tau$ -pseudocompactness.** S. Garcia-Ferreira and H. Ohta gave a construction that was intended to produce a tau-pseudocompact space, which has a regular-closed zero set  $A$  and a regular-closed  $C$ -embedded set  $B$  such that neither  $A$  nor  $B$  is  $\tau$ -pseudocompact. We show that although their sets  $A, B$  are not regular-closed, there are at least two ways to make their construction work to give the desired example.

<http://arxiv.org/abs/math.GN/0505516>

*Jerry E. Vaughan*

**2.4. Understanding preservation theorems:  $\omega^\omega$ -bounding.** This is an expository note giving Shelah’s proof of the preservation of “proper +  $\omega^\omega$ -bounding” (Theorem 1.12 of Chapter VI of his book *Proper and Improper Forcing*).

<http://arxiv.org/abs/math.LO/0505645>

*Chaz Schlindwein*

**2.5. Classification problems in continuum theory.** We study several natural classes and relations occurring in continuum theory from the viewpoint of descriptive set theory and infinite combinatorics. We provide useful characterizations for the relation of likeness among dendrites and show that it is a bqo with countably many equivalence classes. For dendrites with finitely many branch points the homeomorphism and quasi-homeomorphism classes coincide, and the minimal quasi-homeomorphism classes among dendrites with infinitely many branch points are identified. In contrast, we prove that the homeomorphism relation between dendrites is  $S_\infty$ -universal. It is shown that the classes of trees and graphs are both  $D_2(\Sigma_3^0)$ -complete, the class of dendrites is  $\Pi_3^0$ -complete, and the class of all continua homeomorphic to a graph or dendrite with finitely many branch points is  $\Pi_3^0$ -complete. We also show that if  $G$  is a nondegenerate finitely triangulable continuum, then the class of  $G$ -like continua is  $\Pi_2^0$ -complete.

<http://www.ams.org/journal-getitem?pii=S0002-9947-05-03956-5>

*Riccardo Camerlo, Udayan B. Darji, and Alberto Marcone*

**2.6. Residuality of families of  $F_\sigma$  sets.** We prove that two natural definitions of residuality of families of  $F_\sigma$  sets are equivalent. We make use of the Banach-Mazur game in the proof.

<http://arxiv.org/abs/math.GN/0506379>

*Shingo Saito*

**2.7. First countable, countably compact spaces and the continuum hypothesis.** We build a model of ZFC+CH in which every first countable, countably compact space is either compact or contains a homeomorphic copy of  $\omega_1$  with the order topology. The majority of the paper consists of developing forcing technology that allows us to conclude that our iteration adds no reals. Our results generalize Saharon Shelah's iteration theorems appearing in Chapters V and VIII of *Proper and improper forcing* (1998), as well as Eisworth and Roitman's (1999) iteration theorem. We close the paper with a ZFC example (constructed using Shelah's club-guessing sequences) that shows similar results do not hold for closed pre-images of  $\omega_2$ .

<http://www.ams.org/journal-getitem?pii=S0002-9947-05-04034-1>

*Todd Eisworth and Peter Nyikos*

**2.8. Game approach to universally Kuratowski-Ulam spaces.** We consider a variant of the open-open game. A topological characterization of I-favorable spaces is used to show that the hyperspace over I-favorable space is I-favorable. The main result is that every I-favorable space is universally Kuratowski-Ulam.<sup>1</sup>

*Andrzej Kucharski and Szymon Plewik*

**2.9. Small Valdivia compact spaces.** We prove a preservation theorem for the class of Valdivia compact spaces, which involves inverse sequences of "simple" retractions. Consequently, a compact space of weight  $\leq \aleph_1$  is Valdivia compact iff it is

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<sup>1</sup>A space  $Y$  is universally Kuratowski-Ulam if the Kuratowski-Ulam Theorem holds in  $X \times Y$  for every space  $X$ .

the limit of an inverse sequence of metric compacta whose bonding maps are retractions. As a corollary, we show that the class of Valdivia compacta of weight  $\leq \aleph_1$  is preserved both under retractions and under open 0-dimensional images. Finally, we characterize the class of all Valdivia compacta in the language of category theory, which implies that this class is preserved under all continuous weight preserving functors.

<http://arxiv.org/abs/math.GN/0507062>

*Wiesław Kubiś and Henryk Michalewski*

**2.10. Canonical forms of Borel functions on the Milliken space.** The goal of this paper is to canonize Borel measurable mappings  $\Delta: \Omega^\omega \rightarrow \mathbb{R}$ , where  $\Omega^\omega$  is the Milliken space, i.e., the space of all increasing infinite sequences of pairwise disjoint nonempty finite sets of  $\omega$ . This main result is a common generalization of a theorem of Taylor and a theorem of Prömel and Voigt.

<http://www.ams.org/journal-getitem?pii=S0002-9947-05-04000-6>

*Olaf Klein and Otmar Spinas*

**2.11. Complete analytic equivalence relations.** We prove that various concrete analytic equivalence relations arising in model theory or analysis are complete, i.e. maximum in the Borel reducibility ordering. The proofs use some general results concerning the wider class of analytic quasi-orders.

<http://www.ams.org/journal-getitem?pii=S0002-9947-05-04005-5>

*Alain Louveau and Christian Rosendal*

**2.12. The Number of Near-Coherence Classes of Ultrafilters is Either Finite or  $2^c$ .** We prove that the number of near-coherence classes of non-principal ultrafilters on the natural numbers is either finite or  $2^c$ . Moreover, in the latter case the Stone-Čech compactification  $\beta\mathbb{N}$  of  $\mathbb{N}$  contains a closed subset  $C$  consisting of  $2^c$  pairwise non-nearly-coherent ultrafilters. We obtain some additional information about such closed sets under certain assumptions involving the cardinal characteristics  $\mathfrak{u}$  and  $\mathfrak{d}$ .

Applying our main result to the Stone-Čech remainder  $\beta\mathbb{R}_+ \setminus \mathbb{R}_+$  of the half-line  $\mathbb{R}_+ = [0, \infty)$  we obtain that the number of composants of  $\beta\mathbb{R}_+ \setminus \mathbb{R}_+$  is either finite or  $2^c$ .

*Taras Banach and Andreas Blass*

**2.13. The complexity of recursion theoretic games.** We show that some natural games introduced by Lachlan in 1970 as a model of recursion theoretic constructions are undecidable, contrary to what was previously conjectured. Several consequences are pointed out; for instance, the set of all  $\Pi_2$ -sentences that are uniformly valid in the lattice of recursively enumerable sets is undecidable. Furthermore we show that these games are equivalent to natural subclasses of effectively presented Borel games.

<http://www.ams.org/journal-getitem?pii=S0002-9947-05-04074-2>

*Martin Kummer*

2.14. **Cosmic dimension.** Martin's Axiom for  $\sigma$ -centered partial orders implies that there is a cosmic space with non-coinciding dimensions.

<http://arxiv.org/abs/math.GN/0509097>

*Alan Dow and Klaas Pieter Hart*

2.15. **There is no categorical metric continuum.** We show there is no categorical metric continuum. This means that for every metric continuum  $X$  there is another metric continuum  $Y$  such that  $X$  and  $Y$  have (countable) elementarily equivalent bases but  $X$  and  $Y$  are not homeomorphic. As an application we show that the chainability of the pseudoarc is not a first-order property of its lattice of closed sets.

<http://arxiv.org/abs/math.GN/0509099>

*Klaas Pieter Hart*

### 3. ON A CONJECTURE AND A PROBLEM OF HUREWICZ

A set of reals  $X$  has *Menger's property* (1924) if no continuous image of  $X$  in  ${}^{\mathbb{N}}\mathbb{N}$  is cofinal with respect to  $\leq^*$ . It has the formally stronger *Hurewicz' property* (1925) if every continuous image of  $X$  in  ${}^{\mathbb{N}}\mathbb{N}$  is bounded.  $\sigma$ -compactness implies Hurewicz' property, which implies Menger's. Both Menger and Hurewicz conjectured that their property characterizes  $\sigma$ -compactness, and for a long time only consistent counter-examples were known. Hurewicz (1927) also posed the problem whether there is  $X \subseteq \mathbb{R}$  which is Hurewicz but not Menger. The problem was raised again by Bukovský and Haleš (2003).

Fremlin-Miller (1988) and then Just-Miller-Scheepers-Szeptycki (1996) gave a dichotomic existential argument refuting the Conjectures in ZFC. Using the Michael topological technique, Chaber-Pol (2002) improved the dichotomic argument and essentially solved the Hurewicz Problem, alas in an existential manner.

Barotszyński-Tsaban (2002) gave two explicit counter-examples to the conjectures using two specialized constructions. Tsaban-Zdomsky (2005) generalize both constructions and solve the Hurewicz Problem constructively by considering scales with respect to semifilters (collections of infinite subsets of  $\mathbb{N}$  closed under almost supersets). Working in  $P(\mathbb{N})$  (which is like  ${}^{\mathbb{N}}\{0, 1\}$ ): For each feeble semifilter  $\mathcal{F}$  and each  $\mathcal{F}$ -scale  $S$ , all finite powers of  $X = S \cup [\mathbb{N}]^{<\aleph_0}$  are Hurewicz and not  $\sigma$ -compact. Viewed appropriately as a subset of  $\mathbb{R}$ , the field generated by  $X$  is Hurewicz, universally null, and universally meager. The Hurewicz problem is solved by using the semifilter  $\mathcal{F} = P_{\infty}(\mathbb{N})$ , and choosing the  $\mathcal{F}$ -scale's points such that (the enumerations of) their complements form an unbounded set. To carry this, descriptive set theoretic properties of semifilters are used. When  $\mathfrak{d}$  is regular, subfields of  $\mathbb{R}$  are constructed which are Menger but not Hurewicz. This implies (in a dichotomic manner) that there always are such fields.

All results can be viewed as dealing with the Ramsey theory of open covers.

The work of Tsaban-Zdomsky, named *Scales, fields, and a problem of Hurewicz*, is available at

<http://arxiv.org/abs/math.GN/0507043>

*Boaz Tsaban*

## 4. PROBLEM OF THE ISSUE

The most interesting problem which arises from the work described in Section 3 is the following.

**Problem 4.1.** *Does there exist (in ZFC) a set of reals  $X$  of cardinality  $\mathfrak{d}$  such that all finite powers of  $X$  have Menger's property  $\mathbf{U}_{fin}(\mathcal{O}, \mathcal{O})$ ?*

We know that the answer is “Yes” when  $\mathfrak{d}$  is regular. If the answer is positive, then an explicit (non-dichotomic) construction of such a set would be even more interesting.

*Boaz Tsaban*

## 5. PROBLEMS FROM EARLIER ISSUES

**Issue 1.** *Is  $\left(\frac{\Omega}{\Gamma}\right) = \left(\frac{\Omega}{\mathbb{T}}\right)$ ?*

**Issue 2.** *Is  $\mathbf{U}_{fin}(\Gamma, \Omega) = \mathbf{S}_{fin}(\Gamma, \Omega)$ ? And if not, does  $\mathbf{U}_{fin}(\Gamma, \Gamma)$  imply  $\mathbf{S}_{fin}(\Gamma, \Omega)$ ?*

**Issue 4.** *Does  $\mathbf{S}_1(\Omega, \mathbb{T})$  imply  $\mathbf{U}_{fin}(\Gamma, \Gamma)$ ?*

**Issue 5.** *Is  $\mathfrak{p} = \mathfrak{p}^*$ ? (See the definition of  $\mathfrak{p}^*$  in that issue.)*

**Issue 6.** *Does there exist (in ZFC) an uncountable set satisfying  $\mathbf{S}_1(\mathcal{B}_\Gamma, \mathcal{B})$ ?*

**Issue 8.** *Does  $X \notin \text{NON}(\mathcal{M})$  and  $Y \notin \mathbf{D}$  imply that  $X \cup Y \notin \text{COF}(\mathcal{M})$ ?*

**Issue 9.** *Is  $\text{Split}(\Lambda, \Lambda)$  preserved under taking finite unions?*

*Partial solution.* Consistently yes (Zdomsky). Is it “No” under CH? □

**Issue 10.** *Is  $\text{cov}(\mathcal{M}) = \mathfrak{o}\mathfrak{d}$ ? (See the definition of  $\mathfrak{o}\mathfrak{d}$  in that issue.)*

**Issue 11.** *Does  $\mathbf{S}_1(\Gamma, \Gamma)$  always contain an element of cardinality  $\mathfrak{b}$ ?*

**Issue 12.** *Could there be a Baire metric space  $M$  of weight  $\aleph_1$  and a partition  $\mathcal{U}$  of  $M$  into  $\aleph_1$  meager sets where for each  $\mathcal{U}' \subset \mathcal{U}$ ,  $\bigcup \mathcal{U}'$  has the Baire property in  $M$ ?*

**Previous issues.** The first issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, on <http://arxiv.org/abs/math.GN/x>, where  $x$  is 0301011, 0302062, 0303057, 0304087, 0305367, 0312140, 0401155, 0403369, 0406411, 0409072, 0412305, 0503631, and 0508563, respectively, for issues number 1 to 13.

**Contributions.** Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in  $\text{\LaTeX}$ . The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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