

SPM BULLETIN

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1. EDITOR'S NOTE

Please mark your calendars: 19–22 December 2005 are the days of the coming large workshop on Selection Principles in Mathematics. The current list of participants (to be available in the Workshop's homepage soon) promises that this will be an exceptionally interesting workshop. We hope to see many of the readers of this bulletin there. See Section 2.1.

In Section 2.2 we announce another, very close (by time and theme), fascinating workshop, to be held in 24–31 July 2005.

This issue also contains, as usual, research announcements.

Contributions to the next issue are, as always, welcome.

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2. WORKSHOPS ON SPM THEMES

2.1. **Second workshop on Coverings, Selections and Games in Topology (SPM05).** Lecce, Italy, 19–22 December 2005.

The study of Selection Principles in Mathematics has experienced rapid expansion during the past few years with a large number of mathematicians contributing to the area, and entering the area. The combination of classical and modern methods has led to fascinating breakthroughs and to complete solutions of some of the oldest open problems (1920's and 1930's) in the field, and a large number of new problems covering a variety of topics in mathematics have been identified. Since 2001 several strong Ph.D. theses were devoted exclusively to topics in Selection Principles in Mathematics.

Though Selection Principles in Mathematics had its original beginnings mostly in the study of covering properties of topological spaces that were introduced by Menger (1924), Hurewicz (1925), Rothberger (1937) and Sierpinski (1937), the field has become vastly wider. There are currently several well-defined focus areas in Selection Principles in Mathematics, including:

- (1) Distributivity properties in Boolean algebras
- (2) Combinatorial properties of filters on the natural numbers
- (3) Boundedness properties in topological groups
- (4) Closure- and convergence- properties in function spaces
- (5) Combinatorial cardinal characteristics of the continuum
- (6) Selective screenability and covering dimension
- (7) Covering properties of topological spaces.

The aim of the workshop is to survey current directions in the field through a number of plenary talks and to learn about current results and open problems in this area through a number of shorter contributed talks.

A tentative list of plenary speakers includes:

- Liljana Babinkostova (Boise State University)
- Taras Banach (Lviv University)
- Lev Bukovsky (P. J. Safarik University)
- Filippo Camoroto (University of Messina)
- Ljubisa Kocinac (University of Nis)
- Giuseppe Di Maio (Second University of Napoli)
- Heike Mildenberger (Kurt Gödel Research Center for Mathematical Logic)
- Arnold Miller (University of Wisconsin)
- Masami Sakai (Kanagawa University)
- Marion Scheepers (Boise State University)
- Boaz Tsaban (Weizmann Institute of Science)
- Lubomyr Zdomsky (Lviv University)

The registration fee for the conference is 50 Euros. There is limited financial support available for participation. It could cover the registration fee, accommodation or meals. People who have no other financial support for participating in the conference should apply **before the end of June, 2005**, to the Organizing

Committee for financial support. Direct all applications to Professor Cosimo Guido (cosimo.guido@unile.it).

Information on travelling to Lecce and on accommodations will become available in the near future at the workshop web sites currently under construction. The url's are:

http://www.matematica.unile.it/mostra_avviso.asp?n=136

<http://diamond.boisestate.edu/~spm/Lecce2/index.htm>

Topology Atlas is generously providing abstract services for this conference. Please submit abstracts at the following web-site:

<http://atlas-conferences.com/cgi-bin/abstract/submit/caqh-01>

Please note that the deadline for submitting abstracts is **October 31, 2005**.

Submitted abstracts can be viewed at

<http://atlas-conferences.com/cgi-bin/abstract/caqh-01>

The current list of sponsors for the meeting includes: The University of Lecce, Department of Mathematics "E. De Giorgi" – University of Lecce, and Topology Atlas.

L. Babinkostova, C. Guido, L. Kocinac, M. Scheepers, and B. Tsaban

2.2. Analysis and Descriptive Set Theory Workshop. 24–31 July 2005, Banach Center, Bedlewo, Poland.

Descriptive set theory: Effective methods, equivalence relations. Speakers: J. Cichoń (Wrocław, Poland) and S. Solecki (Urbana-Champaign, USA).

Recent applications of descriptive set theory emphasize investigation of definable equivalence relations in various mathematical contexts (as opposed to the study of definable sets) and usage of effective (recursive theoretic) methods. At the root of this development lie, on the one hand, the theorem of Silver that each co-analytic equivalence relation on a Polish space either has countably many classes or there exists a perfect set of inequivalent elements and, on the other hand, the effective descriptive set theoretic proof of this theorem due to Harrington. Extensions of Silver's theorem and of its method of proof have found numerous applications in Polish group actions, in certain classification problems in algebra and topology, in continuum theory, and in real function theory, to name only a few.

In the workshop, we propose to present Harrington's proof of Silver's theorem along with a survey of a broader mathematical background of this result. The talks will be aimed at graduate students interested in applications of descriptive set theory. Prior exposure to basic facts of classical descriptive set theory will make motivation of some of the material clearer.

Here is the plan of the first half of the workshop.

- (1) Rudiments of recursion theory through Kleene's recursion theorem.
- (2) Survey of basic theorems of classical descriptive set theory (Borel, analytic, co-analytic sets, operation \mathcal{A} , Nikodym's theorem on preservation of Baire property by operation \mathcal{A} , Mycielski's theorem on perfect independent sets for meager relations).

- (3) Kleene's classes Σ_1^0 , Π_1^0 , Σ_1^1 , Π_1^1 , good universal sets.
- (4) Π_1^1 sets and their representation via well-orderings, boundedness, reflection theorem, coding Δ_1^1 sets, Gandy's basis theorem for Σ_1^1 sets.
- (5) The Gandy-Harrington topology and the proof of Silver's theorem.

Analysis: typical functions, level sets structure. Speakers: U. B. Darji (Louisville, USA) and M. Morayne (Wrocław, Poland)

The purpose of this course is to study properties of smooth functions in terms of their level sets. We will first recall the Baire Category Theorem and the celebrated Banach Theorem that a typical continuous function on $[0, 1]$ is nowhere differentiable. Using the Baire Category theorem, we will also prove the existence and the abundance of otherwise complicated objects in various branches of analysis and topology. However, the main thrust is to study the behavior of a typical smooth function in terms of its level sets (inverse images of points) structure. This topic was opened by a well known theorem of Bruckner and Garg describing the level sets of typical continuous functions defined on I . The proof of this theorem will be presented as well as the proofs of recent results concerning level sets of typical C^n functions, $1 \leq n \leq \infty$. We will also discuss level sets structure of a typical continuous function from S^2 , the 2-sphere, into I . This is more of a topological result which shows that objects such as pseudoarcs, pseudocircles and Lakes of Wada continuum appear naturally in a typical map from S^2 into I .

Our second topic is the "worst case behavior" of the level sets structure of smooth functions. This will involve geometric measure theory and descriptive set theory. We will prove recent results which describe how to "parametrize" Hausdorff dimension of analytic sets by smooth functions. As a simple corollary to these results, it will follow that there is a C^∞ function $f : I \rightarrow I$ such that the set of points where the level sets of f is uncountable is large in terms of measure and complicated descriptive set theoretically. This is counter intuitive to the belief held by some that " *C^∞ functions are more or less real analytic.*" We will discuss many interesting and open question.

The indicatrix of a function is the function assigning to a point in the range of a function the cardinality of its level set. The behaviour of indicatrices of Lebesgue measurable functions and Borel measurable functions will be described. This seems to be a topic of still many open and interesting questions.

Here is the plan of the second half of the workshop:

- (1) Baire's category theorem and Banach's proof of the existence of a nowhere differentiable function.
- (2) Pompeiu derivatives and Köpcke (i.e. everywhere differentiable and nowhere monotone) functions. Weil's proof of the existence of Köpcke functions using Baire's category method.
- (3) Level sets of 'typical' continuous functions in one dimension - the theorem of Bruckner and Garg.
- (4) Level sets of 'typical' C^n functions in one dimension and of typical continuous functions in two dimensions - some newer developments.

(5) An introduction to Hausdorff measures. The description of collections of perfect and uncountable level sets for C^n functions.

(6) Characterizations of indicatrices of Lebesgue and Borel measurable functions.

(The results in sections 4,5,6 are due to D'Aniello, Buczolic, Komisarski, Milewski, Michalewski, Ryll-Nardzewski and to the speakers.)

Organizational and additional details. See

<http://www.im.pwr.wroc.pl/~cichon/Bedlewo/>

For additional information contact morayne@im.pwr.wroc.pl

Jacek Cichoń

3. RESEARCH ANNOUNCEMENTS

3.1. Cardinal restrictions on some homogeneous compacta. We give restrictions on the cardinality of compact Hausdorff homogeneous spaces that do not use other cardinal invariants, but rather covering and separation properties. In particular, we show that it is consistent that every hereditarily normal homogeneous compactum is of cardinality \mathfrak{c} . We introduce property $wD(\kappa)$, intermediate between the properties of being weakly κ -collectionwise Hausdorff and strongly κ -collectionwise Hausdorff, and show that if X is a compact Hausdorff homogeneous space in which every subspace has property $wD(\aleph_1)$, then X is countably tight and hence of cardinality $\leq 2^{\mathfrak{c}}$. As a corollary, it is consistent that such a space X is first countable and hence of cardinality \mathfrak{c} . A number of related results are shown and open problems presented.

<http://www.ams.org/journal-getitem?pii=S0002-9939-05-07861-5>

Istvan Juhasz, Peter Nyikos, and Zoltan Szentmiklossy

3.2. Filters: Topological congruence relations on groups. A filter \mathcal{F} on a group G is a T -filter if there is a Hausdorff group topology τ on G such that $\mathcal{F} \xrightarrow{\tau} 0$. This notion can be specialized for sequences, in which case we say that $\{a_n\}$ is a T -sequence. In this paper, T -filters and T -sequences are studied. We characterize T -filters in non-abelian groups, show that certain filters can be interpreted as topological extensions of the notion of kernel (i.e., normal subgroup, congruence relation), and provide several sufficient conditions for a sequence in an abelian group to be a T -sequence. As an application, special sequences in the Prüfer groups $\mathbb{Z}(p^\infty)$ are investigated. We prove that for $p \neq 2$, there is a Hausdorff group topology τ on $\mathbb{Z}(p^\infty)$ that is neither maximally nor minimally almost periodic—in other words, the von Neumann radical $\mathbf{n}(\mathbb{Z}(p^\infty), \tau)$ is a non-trivial finite subgroup. In particular, $\mathbf{n}(\mathbf{n}(\mathbb{Z}(p^\infty), \tau)) \subsetneq \mathbf{n}(\mathbb{Z}(p^\infty), \tau)$.

<http://arxiv.org/abs/math.GN/0504003>

Gábor Lukács

3.3. Inverse Limits and Function Algebras. Assuming Jensen's principle diamond, there is a compact Hausdorff space X which is hereditarily Lindelöf, hereditarily separable, and connected, such that no closed subspace of X is both perfect and totally disconnected. The Proper Forcing Axiom implies that there is no such

space. The diamond example also fails to satisfy the CSWP (the complex version of the Stone-Weierstrass Theorem). This space cannot contain the two earlier examples of failure of the CSWP, which were totally disconnected – specifically, the Cantor set (W. Rudin) and $\beta\mathbb{N}$ (Hoffman and Singer).

<http://arxiv.org/abs/math.GN/0504214>

Joan E. Hart and Kenneth Kunen

3.4. Ultrafilters and partial products of infinite cyclic groups. We consider, for infinite cardinals κ and $\alpha \leq \kappa^+$, the group $\Pi(\kappa, < \alpha)$ of sequences of integers, of length κ , with non-zero entries in fewer than alpha positions. Our main result tells when $\Pi(\kappa, < \alpha)$ can be embedded in $\Pi(\lambda, < \beta)$. The proof involves some set-theoretic results, one about families of finite sets and one about families of ultrafilters.

<http://arxiv.org/abs/math.LO/0504199>

Andreas Blass and Saharon Shelah

3.5. More on regular reduced products. The authors show, by means of a finitary version $\square_{\lambda, D}^{fin}$ of the combinatorial principle \square_{λ}^{b*} , the consistency of the failure, relative to the consistency of supercompact cardinals, of the following: For all regular filters D on a cardinal λ , if M_i and N_i are elementarily equivalent models of a language of size $\leq \lambda$, then the second player has a winning strategy in the Ehrenfeucht-Fraïssé game of length λ^+ on $\prod_i M_i/D$ and $\prod_i N_i/D$. If in addition $2^\lambda = \lambda^+$ and $i < \lambda$ implies $|M_i| + |N_i| \leq \lambda^+$, this means that the ultrapowers are isomorphic.

<http://arxiv.org/abs/math.LO/0504200>

Juliette Kennedy and Saharon Shelah

3.6. Consistency of “the ideal of null restricted to some A is κ -complete not κ^+ -complete, κ weakly inaccessible and $\text{cov}(\mathcal{M}) = \aleph_1$ ”. In this note we answer the following question of Grinblat: Is it consistent that for some set A , $\text{cov}(\text{null} \upharpoonright A) = \lambda$ is a weakly inaccessible cardinal (so A is not null) while $\text{cov}(\text{meager})$ is small, say it is \aleph_1 .

<http://arxiv.org/abs/math.LO/0504201>

Saharon Shelah

3.7. On removing one point from a compact space. If B is a compact space and $B \setminus \{pt\}$ is Lindelöf then $B^k \setminus \{pt\}$ is star-Lindelöf for every cardinality k . If $B \setminus \{pt\}$ is compact then $B^k \setminus \{pt\}$ is discretely star-Lindelöf. In particular, this gives new examples of Tychonoff discretely star-Lindelöf spaces with unlimited extent.

<http://arxiv.org/abs/math.GN/0504325>

Gady Kozma

4. PROBLEM OF THE ISSUE

Since this issue is sent out earlier than planned, it will contain no *Problem of the Issue* this time. The next section contains many problems which are still open, so that the reader may wish to solve any of them instead.

5. PROBLEMS FROM EARLIER ISSUES

In this section we list the still open problems among the past problems posed in the *SPM Bulletin* (in the section *Problem of the month/issue*). For definitions, motivation and related results, consult the corresponding issue.

For conciseness, we make the convention that all spaces in question are zero-dimensional, separable metrizable spaces.

Issue 1. *Is $(\frac{\Omega}{\Gamma}) = (\frac{\Omega}{\Gamma})$?*

Issue 2. *Is $U_{fin}(\Gamma, \Omega) = S_{fin}(\Gamma, \Omega)$? And if not, does $U_{fin}(\Gamma, \Gamma)$ imply $S_{fin}(\Gamma, \Omega)$?*

Issue 4. *Does $S_1(\Omega, \Gamma)$ imply $U_{fin}(\Gamma, \Gamma)$?*

Issue 5. *Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of \mathfrak{p}^* in that issue.)*

Issue 6. *Does there exist (in ZFC) an uncountable set satisfying $S_1(\mathcal{B}_\Gamma, \mathcal{B})$?*

Issue 8. *Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \text{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?*

Issue 9. *Is $\text{Split}(\Lambda, \Lambda)$ preserved under taking finite unions?*

Partial solution. Consistently yes (Zdomsky). Is it “No” under CH? □

Issue 10. *Is $\text{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of \mathfrak{od} in that issue.)*

Issue 11. *Does $S_1(\Gamma, \Gamma)$ always contain an element of cardinality \mathfrak{b} ?*

Issue 12. *Could there be a Baire metric space M of weight \aleph_1 and a partition \mathcal{U} of M into \aleph_1 meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M ?*

Previous issues. The first issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, on <http://arxiv.org/abs/math.GN/x>, where x is 0301011, 0302062, 0303057, 0304087, 0305367, 0312140, 0401155, 0403369, 0406411, 0409072, 0412305, and 0503631, respectively, for issues number 1 to 12.

Contributions. Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in \LaTeX . The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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