

SPM BULLETIN

ISSUE NUMBER 12: March 2005 CE

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1. EDITOR'S NOTE

1. Proceedings of first SPM Workshop. The proceedings of the first workshop on *Coverings, Selections, and Games in Topology* (Lecce, Italy, June 27–29, 2002) were published as a special issue of **Note di Matematica** (Volume 22, Issue 2 (2003)), and are now also available online at:

http://siba2.unile.it/ejournals/search/issue.php?single_pub=1&pubid=1&id=342&barcol=000033&recs=10

(Alternatively, go to <http://siba2.unile.it/notemat/>, choose “Contents, Abstracts, Full text articles”, then under the year 2003, choose “vol. 22, issue 2”). In [§2 below] we quote the editors’ preface to the volume.

The next SPM Workshop is planned for the coming December, details are yet to come.

2. Mathematical breakthroughs. Justin Moore announces in the Mathematics ArXiv a sequence of astonishing results. Liljana Babinkostova has solved an 1938 problem of Rothberger, and established a game theoretic characterization of countable dimensionality. These and other beautiful mathematical results are announced below.

3. Mathematics ArXiv. Elliot Peal, editor of the *Topology Atlas*, recommends storing preprints in the ArXiv. We quote his message from the bulletin *Topology News*:

Topology Atlas maintains a preprint server. Our preprint server was most active in 1996 and 1997 and nearly all of our preprints have been published elsewhere by now. We suggest that you use the mathematics arXiv ([arXiv.org](http://arxiv.org)) or its Front (front.math.ucdavis.edu) for storing and finding preprints. The arXiv is a powerful and stable resource for distributing results in active research communities. Upon request, we can help you archive all your TeX files in the arXiv.

We support this recommendation, and also recommend that you get subscribed to the mailing lists of the arXiv, the categories of interest to readers of the *SPM Bulletin* are probably those of General Topology and Logic. Details on subscription etc. are available at

<http://arxiv.org/>

3. Change of name. We were informed by an anonymous referee that the name \mathfrak{o} used in Issue 10 of the *SPM Bulletin* (and its subsequent issues) for the *o*-diagonalization number is already used for Leathrum’s *off-branching* number [8]. We therefore move to use \mathfrak{od} for the new cardinal.

Contributions to the next issue are, as always, welcome.

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2. PROCEEDINGS OF THE FIRST WORKSHOP ON COVERINGS, SELECTIONS, AND GAMES IN TOPOLOGY

The first Workshop on Coverings, Selections and Games in Topology was held in Lecce, Italy, on June 27-29, 2002 and organized by the Department of Mathematics of Lecce University.

The aim of this workshop was to consider development and activities over the last few years in a quickly growing field of Mathematics known under the name Selection Principles and to discuss its relationships with other areas of Mathematics, such as (generalized) Ramsey theory, infinite game theory, combinatorial cardinals, function spaces, hyperspaces, uniform spaces, topological algebras and so on.

The work was organized in such a way as to have plenary 50 minute lectures on general aspects as well as on some specific topics and 25 minute contributed talks. Each session was ended by a discussion on presented results. The conference has been closed by a Round Table. The goal was to consider and discuss further common research activities, open problems in the field and the organization of new future meetings on the same topic. It was decided to publish the Proceedings of the meeting as a special issue of *Note di Matematica* following the usual referee procedure established by the journal. In particular, it has been decided the Proceedings to contain two survey papers: the first one opens this volume and presents general aspects of the theory of selection principles with recent progress in this area and with directions of further investigation; the second one, which closes the volume, is devoted to the presentation and discussion of open problems. Special thanks are due to the authors of these surveys.

Also, it was planned to edit the SPM Bulletin, an electronic bulletin dedicated to the field with announcements of new results as well as with open problems.

We hope that these Proceedings which contain some of the papers presented at the Workshop will help to attract new researchers and stimulate the investigation in this field of Mathematics.

Cosimo Guido, Ljubisa D. R. Kočinac, and Marion Scheepers

2.1. Contents of the proceedings volume.

- (1) C. Guido, L. D. R. Kočinac, M. Scheepers:
Preface (p. 1).
- (2) M. Scheepers:
Selection principles and covering properties in Topology (p. 3).
- (3) M. Sakai:
The Pytkeev property and the Reznichenko property in function spaces (p. 43).
- (4) B. Tsaban:
Selection principles and the Minimal Tower problem (p. 53).
- (5) T. Weiss, B. Tsaban:
Topological diagonalizations and Hausdorff dimension (p. 83).
- (6) D. Leseberg:
Symmetrical extensions and generalized nearness (p. 93).
- (7) M. Caldas, D. N. Georgiou, S. Jafari, T. Noiri:
More on δ -semiopen sets (p. 113).
- (8) L. D. R. Kocinac:
Selection principles in uniform spaces (p. 127).
- (9) S. D. Iliadis:
Saturated classes of bases (p. 141).

- (10) S. E. Han:
Generalized digital (k_0, k_1) -homeomorphism (p. 157).
- (11) L. Babinkostova, M. Scheepers:
Combinatorics of open covers (IX): Basis properties (p. 167).
- (12) B. Tsaban:
Selection principles in mathematics: A milestone of open problems (p. 179).

3. ADDITIONAL RESEARCH ANNOUNCEMENTS

3.1. A five element basis for the uncountable linear orders. In this paper I will show that it is relatively consistent with the usual axioms of mathematics (ZFC) together with a strong form of the axiom of infinity (the existence of a supercompact cardinal) that the class of uncountable linear orders has a five element basis. In fact such a basis follows from the Proper Forcing Axiom, a strong form of the Baire Category Theorem. The elements are $X, \omega_1, \omega_1^*, C, C^*$ where X is any suborder of the reals of cardinality \aleph_1 and C is any Countryman line. This confirms a longstanding conjecture of Shelah.

<http://arxiv.org/abs/math.LO/0501525>
Justin Tatch Moore

3.2. Set mapping reflection. In this note we will discuss a new reflection principle which follows from the Proper Forcing Axiom. The immediate purpose will be to prove that the bounded form of the Proper Forcing Axiom implies both that $2^{\aleph_0} = \aleph_2$ and that $L(P(\aleph_1))$ satisfies the Axiom of Choice. It will also be demonstrated that this reflection principle implies that combinatorial principle $\square(\kappa)$ fails for all regular $\kappa > \aleph_1$.

<http://arxiv.org/abs/math.LO/0501526>
Justin Tatch Moore

3.3. The Proper Forcing Axiom, Prikry forcing, and the Singular Cardinals Hypothesis. The purpose of this paper is to present some results which suggest that the Singular Cardinals Hypothesis follows from the Proper Forcing Axiom. What will be proved is that a form of simultaneous reflection follows from the Set Mapping Reflection Principle, a consequence of PFA. While the results fall short of showing that MRP implies SCH, it will be shown that MRP implies that if SCH fails first at κ then every stationary subset of $S_{\kappa^+}^\omega = \{\alpha < \kappa^+ : \text{cf}(\alpha) = \omega\}$ reflects. It will also be demonstrated that MRP always fails in a generic extension by Prikry forcing.

<http://arxiv.org/abs/math.LO/0501527>
Justin Tatch Moore

3.4. A solution to the L space problem and related ZFC constructions. In this paper I will construct a non-separable hereditarily Lindelof space (L space) without any additional axiomatic assumptions. I will also show that there is a function $f : [\omega_1]^2 \rightarrow \omega_1$ such that if $A, B \subseteq \omega_1$, are uncountable and $\xi < \omega_1$, then there are $\alpha < \beta$ in A and B respectively with $f\{\alpha, \beta\} = \xi$.

Previously it was unknown whether such a function existed even if ω_1 was replaced by 2. Finally, I will prove that there is no basis for the uncountable regular Hausdorff spaces of cardinality \aleph_1 . Each of these results gives a strong refutation of a well known and longstanding conjecture. The results all stem from the analysis of oscillations of coherent sequences $\langle e_\alpha : \alpha < \omega_1 \rangle$ of finite-to-one functions. I expect that the methods presented will have other applications as well.

<http://arxiv.org/abs/math.GN/0501524>

Justin Tatch Moore

3.5. Countable Tightness, Elementary Submodels and Homogeneity. We show (in ZFC) that the cardinality of a compact homogeneous space of countable tightness is no more than the size of the continuum.

<http://arxiv.org/abs/math.GN/0501076>

Ramiro de la Vega

3.6. No transcendence basis of \mathbb{R} over \mathbb{Q} can be an analytic set. It has been proved by Sierpiński that no linear basis of \mathbb{R} over \mathbb{Q} can be an analytic set. Here we show that the same assertion holds by replacing “linear basis” with “transcendence basis”. Furthermore, it is demonstrated that purely transcendental subfields of \mathbb{R} generated by Borel bases of the same cardinality are Borel isomorphic (as fields). Following Mauldin’s arguments, we also indicate, for each ordinal α such that $1 \leq \alpha < \aleph_1$ ($2 \leq \alpha < \aleph_1$), the existence of subfields of \mathbb{R} of exactly additive (multiplicative, ambiguous) class α in \mathbb{R} .

Enrico Zoli

3.7. Two properties of $C_p(X)$ weaker than Fréchet Urysohn property. For a Tychonoff space X , we denote by $C_p(X)$ the space of all real-valued continuous functions on X with the topology of pointwise convergence. In this paper, we study κ -Fréchet Urysohn property and weak Fréchet Urysohn property of $C_p(X)$. Our main results are that:

- (1) $C_p(X)$ is κ -Fréchet Urysohn iff X has property $(\kappa\text{-FU})$ (i.e. every pairwise disjoint sequence of finite subsets of X has a strongly point-finite subsequence), in particular a Baire space $C_p(X)$ is κ -Fréchet Urysohn;
- (2) among separable metrizable spaces, every λ -space has property $(\kappa\text{-FU})$ and every space having property $(\kappa\text{-FU})$ is always of the first category;
- (3) every analytic space has the ω -grouping property, hence for every analytic space X , $C_p(X)$ is weakly Fréchet Urysohn.

Masami Sakai

3.8. Some partition properties for measurable colourings of $(\aleph_1)^2$. We construct a measure on $(\aleph_1)^2$ over the ground model in the forcing extension of a measure algebra, and investigate when measure theoretic properties of some measurable colouring of $(\aleph_1)^2$ imply the existence of an uncountable subset of \aleph_1 whose square is

homogeneous. This gives a new proof of the fact that, under a suitable axiomatic assumption, there are no Souslin (\aleph_1, \aleph_1) gaps in the Boolean algebra $L^0(\nu)/Fin$ when ν is a separable measure.

To appear in: *Proceedings of the Kyoto conference on Forcing Method and Large Cardinals, 2004.*

<http://arxiv.org/abs/math.LO/0501421>

James Hirschorn

3.9. Potential theory and forcing. We isolate a combinatorial property of capacities leading to a construction of proper forcings. Then we show that many classical capacities such as the Newtonian capacity satisfy the property.

<http://arxiv.org/abs/math.LO/0502394>

Jindrich Zapletal

3.10. On decompositions of Banach spaces of continuous functions on Mrówka's spaces. It is well known that if K is infinite compact Hausdorff and scattered (i.e., with no perfect subsets), then the Banach space $C(K)$ of continuous functions on K has complemented copies of c_0 , i.e., $C(K) \sim c_0 \oplus X \sim c_0 \oplus c_0 \oplus X \sim c_0 \oplus C(K)$. We address the question if this could be the only type of decompositions of $C(K) \not\sim c_0$ into infinite-dimensional summands for K infinite, scattered. Making a special set-theoretic assumption such as the continuum hypothesis or Martin's axiom we construct an example of Mrówka's space (i.e., obtained from an almost disjoint family of sets of positive integers) which answers positively the above question.

<http://www.ams.org/journal-getitem?pii=S0002-9939-05-07799-3>

Piotr Koszmider

3.11. A note on D -spaces. We introduce notions of nearly good relations and N -sticky modulo a relation as tools for proving that spaces are D -spaces. As a corollary to general results about such relations, we show that $C_p(X)$ is hereditarily a D -space whenever X is a Lindelöf Σ -space. This answers a question of Matveev, and improves a result of Buzyakova, who proved the same result for X compact.

We also prove that if a space X is the union of finitely many D -spaces, and has countable extent, then X is linearly Lindelöf. It follows that if X is in addition countably compact, then X must be compact. We also show that Corson compact spaces are hereditarily D -spaces. These last two results answer recent questions of Arhangel'skii. Finally, we answer a question of van Douwen by showing that a perfectly normal collectionwise-normal non-paracompact space constructed by R. Pol is a D -space.

<http://arxiv.org/abs/math.GN/0503275>

Gary Gruenhage

3.12. Set-theoretic properties of Schmidt's ideal. We study some set-theoretic properties of Schmidt's σ -ideal on \mathbb{R} , emphasizing its analogies and dissimilarities with both the classical σ -ideals on \mathbb{R} of Lebesgue measure zero sets and of Baire first category sets. We highlight the strict analogy between Schmidt's ideal on \mathbb{R} and Mycielski's ideal on $\{0, 1\}^{\mathbb{N}}$.

Marcin Kysiak and Enrico Zoli

3.13. Almost-disjoint coding and strongly saturated ideals. We show that Martin's Axiom plus $\mathfrak{c} = \aleph_2$ implies that there is no $(\aleph_2, \aleph_2, \aleph_0)$ -saturated σ -ideal on ω_1 .

<http://www.ams.org/journal-getitem?pii=S0002-9939-05-07824-X>

Paul B. Larson

4. SELECTIVE SCREENABILITY AND COVERING DIMENSION

Let X be a topological space. In [3] Bing introduced the following notion of *screenability*: For each open cover \mathcal{U} of X there is a sequence $(\mathcal{V}_n : n < \infty)$ such that: For each n , \mathcal{V}_n is a family of pairwise disjoint open sets; for each n , \mathcal{V}_n refines \mathcal{U} and $\cup_{n < \infty} \mathcal{V}_n$ is an open cover of X . In [1] Addis and Gresham introduced the selective version screenability, defined as follows: For each sequence $(\mathcal{U}_n : n < \infty)$ of open covers of X there is a sequence $(\mathcal{V}_n : n < \infty)$ such that: For each n , \mathcal{V}_n is a family of pairwise disjoint open sets; for each n , \mathcal{V}_n refines \mathcal{U}_n and $\cup_{n < \infty} \mathcal{V}_n$ is an open cover of X . It is evident that selective screenability implies screenability.

Selective screenability is an example of the following selection principle which was introduced in [2]: Let S be a set and let \mathcal{A} and \mathcal{B} be families of collections of subsets of the set S . Then $S_c(\mathcal{A}, \mathcal{B})$ denotes the statement that for each sequence $(\mathcal{U}_n : n < \infty)$ of elements of \mathcal{A} there is a sequence $(\mathcal{V}_n : n < \infty)$ such that

- (1) For each n , \mathcal{V}_n is a family of pairwise disjoint sets;
- (2) For each n , \mathcal{V}_n refines \mathcal{U}_n and
- (3) $\cup_{n < \infty} \mathcal{V}_n$ is a member of \mathcal{B} .

With \mathcal{O} denoting the collection of all open covers of topological space X , $S_c(\mathcal{O}, \mathcal{O})$ is selective screenability.

Addis and Gresham noted that countable dimensional metrizable spaces are selectively screenable, and asked if the converse is true. Pol showed in [10] that the answer is no. The author showed that the countable dimensional metric spaces are exactly characterized by a game-theoretic version of selective screenability. The following game, denoted $G_c(\mathcal{A}, \mathcal{B})$, is naturally associated with $S_c(\mathcal{A}, \mathcal{B})$: Players ONE and TWO play as follows: In the n -th inning ONE first chooses \mathcal{O}_n , a member of \mathcal{A} , and then TWO responds with \mathcal{T}_n which is pairwise disjoint and refines \mathcal{O}_n . A play $(\mathcal{O}_1, \mathcal{T}_1, \dots, \mathcal{O}_n, \mathcal{T}_n, \dots)$ is won by TWO if $\cup_{n < \infty} \mathcal{T}_n$ is a member of \mathcal{B} ; else, ONE wins. We can consider versions of different length of this game as follows: For an ordinal number k let $G_c^k(\mathcal{A}, \mathcal{B})$ be the game played as follows: in the l -th inning ($l < k$) ONE first chooses \mathcal{O}_l , a member of \mathcal{A} , and then TWO responds with a pairwise disjoint \mathcal{T}_l which refines \mathcal{O}_l . A play

$$\mathcal{O}_0, \mathcal{T}_0, \dots, \mathcal{O}_l, \mathcal{T}_l, \dots \quad l < k$$

is won by TWO if $\cup_{l < k} \mathcal{T}_l$ is a member of \mathcal{B} ; else, ONE wins. Thus the game $G_c(\mathcal{A}, \mathcal{B})$ is $G_c^\omega(\mathcal{A}, \mathcal{B})$.

The author showed the following:

Theorem 4.1. *Let X be a metric space.*

- (1) If X is countable dimensional, then TWO has a winning strategy in $G_c^\omega(\mathcal{O}, \mathcal{O})$.
- (2) If TWO has a winning strategy in $G_c^\omega(\mathcal{O}, \mathcal{O})$, then X is countable dimensional.

In Pol's example, TWO has a winning strategy in the game $G_c^{\omega+1}(\mathcal{O}, \mathcal{O})$.

Theorem 4.2. *Let X be a metric space. The following are equivalent:*

- (1) If X is n -dimensional then TWO has a winning strategy in $G_c^{n+1}(\mathcal{O}, \mathcal{O})$.
- (2) If TWO has a winning strategy in $G_c^{n+1}(\mathcal{O}, \mathcal{O})$, then X is n -dimensional.

From this Theorem we obtain that metric space X is n -dimensional if, and only if, TWO has a winning strategy in $G_c^{n+1}(\mathcal{O}, \mathcal{O})$ but not in $G_c^n(\mathcal{O}, \mathcal{O})$.

Liljana Babinkostova

5. ON A PROBLEM OF ROTHBERGER AND SIERPINSKI

Let Y be a subspace of the metric space X . Then \mathcal{O}_X denotes the collection of open covers of X , and \mathcal{O}_{XY} denotes the collection of open covers of Y by sets open in X . Let \mathcal{F}_X denote the collection of finite open covers of X .

In 1924 Menger defined in [9] the following basis property, denoted M by Sierpiński: A metric space X has property M if there is for each basis \mathcal{B} of the space a sequence $(B_n : n < \infty)$ from the base such that the diameters of the B_n 's converge to zero, and the B_n 's cover X .

Hurewicz showed in [6] that a metric space X has Menger's basis property M if, and only if, it has $S_{fin}(\mathcal{O}_X, \mathcal{O}_X)$.

According to Rothberger Sierpiński also defined the basis property M' thus: A metric space X has property M' if there is for each basis \mathcal{B} and each sequence $(\epsilon_n : n < \infty)$ of positive real numbers, a sequence $(B_n : n < \infty)$ of elements of \mathcal{B} such that for each n $diam(B_n) < \epsilon_n$, and $\{B_n : n < \infty\}$ covers X .

Let $M'(X, Y)$ denote the version of M' where we require that the sequence of B_n 's cover the subspace Y . (This is the relative version of the Rothberger basis property.)

Rothberger showed in Theorem 7 of [11] that $S_1(\mathcal{O}_X, \mathcal{O}_X)$ implies M' . He posed these problems in [11]:

Problem A: Does M' imply $S_1(\mathcal{O}_X, \mathcal{O}_X)$?

Problem B: What is the relationship between M' and $S_1(\mathcal{F}_X, \mathcal{O}_X)$?

Fremlin and Miller proved in Theorem 6 of [5] that $M \not\Rightarrow S_1(\mathcal{F}_X, \mathcal{O}_X)$, and that $S_1(\mathcal{F}_X, \mathcal{O}_X) \not\Rightarrow M$.

We solve problem A by proving:

Theorem A. For X a metrizable space with $S_{fin}(\mathcal{O}_X, \mathcal{O}_X)$, the following are equivalent:

- (1) $S_1(\mathcal{O}_X, \mathcal{O}_{XY})$.
- (2) $M'(X, Y)$ holds.

Note that if $Y = X$, then the hypothesis $S_{fin}(\mathcal{O}_X, \mathcal{O}_X)$ is not needed, since by Hurewicz's theorem, it follows from the corresponding properties M' or $S_1(\mathcal{O}_X, \mathcal{O}_X)$. And when $Y = X$, Theorem A solves Rothberger's problem, Problem A.

This also gives a solution to Problem B:

Theorem B. $M' \Rightarrow S_1(\mathcal{F}_X, \mathcal{O}_X)$, but $S_1(\mathcal{F}_X, \mathcal{O}_X) \not\Rightarrow M'$.

Liljana Babinkostova

6. PROBLEM OF THE ISSUE

Problem 6.1. *Is the following statement provable in ZFC?*

Let M be a Baire metric space of weight \aleph_1 , $A \subset \mathbb{R}$ a perfectly meager set of cardinality \aleph_1 and $f : M \rightarrow A$ a continuous mapping. Then there is a nonempty open set $U \subset M$ such that f is constant on U .

The statement in Problem 6.1 is true under $MA + \neg CH$ [7]. A positive answer to Problem 6.1 yields a ZFC example of a Banach space which is weak Asplund but whose dual is not weak* fragmentable [7].

Problem 6.2. *Is it consistent with ZFC (using large cardinals) that there is a Baire metric space M of weight \aleph_1 and a partition \mathcal{U} of M into meager sets such that:*

- (1) $|\mathcal{U}| = \aleph_1$, and
- (2) $\bigcup \mathcal{U}'$ has the Baire property in M for each $\mathcal{U}' \subset \mathcal{U}$?

A positive answer to Problem 6.2 implies a negative answer to Problem 6.1. Indeed, let M and \mathcal{U} have the mentioned properties and let A be any perfectly meager subset of \mathbb{R} of cardinality \aleph_1 . Choose a bijection $\varphi : A \rightarrow \mathcal{U}$ and define $g : M \rightarrow A$ by $g(m) = a$ if $m \in \varphi(a)$. Then g has the Baire property and hence there is a residual set $M' \subset M$ with $g|_{M'}$ continuous. Then M' , A and $f = g|_{M'}$ show that the answer to Problem 6.1 is negative.

A positive answer to Problem 6.2 implies that \aleph_1 is measurable in some transitive model of ZFC containing all ordinals [4].

If there is a precipitous ideal on \aleph_1 , there is, by [4] a Baire metric space of weight 2^{\aleph_1} and a partition of it into meager sets satisfying (1) and (2) of Problem 6.2.

Ondrej Kalenda

7. PROBLEMS FROM EARLIER ISSUES

In this section we list the still open problems among the past problems posed in the *SPM Bulletin* (in the section *Problem of the month/issue*). For definitions, motivation and related results, consult the corresponding issue.

For conciseness, we make the convention that all spaces in question are zero-dimensional, separable metrizable spaces.

Issue 1. *Is $(\frac{\Omega}{\Gamma}) = (\frac{\Omega}{T})$?*

Issue 2. *Is $U_{fin}(\Gamma, \Omega) = S_{fin}(\Gamma, \Omega)$? And if not, does $U_{fin}(\Gamma, \Gamma)$ imply $S_{fin}(\Gamma, \Omega)$?*

Issue 4. *Does $S_1(\Omega, T)$ imply $U_{fin}(\Gamma, \Gamma)$?*

Issue 5. *Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of \mathfrak{p}^* in that issue.)*

Issue 6. *Does there exist (in ZFC) an uncountable set satisfying $S_1(\mathcal{B}_\Gamma, \mathcal{B})$?*

Issue 8. *Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \text{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?*

Issue 9. *Is $\text{Split}(\Lambda, \Lambda)$ preserved under taking finite unions?*

Partial solution. Consistently yes (Zdomsky). Is it “No” under CH? □

Issue 10. *Is $\text{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of \mathfrak{od} in that issue.)*

Issue 11. *Does $S_1(\Gamma, \Gamma)$ always contain an element of cardinality \mathfrak{b} ?*

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Previous issues. The first issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, on <http://arxiv.org/abs/math.GN/x>, where x is 0301011, 0302062, 0303057, 0304087, 0305367, 0312140, 0401155, 0403369, 0406411, 0409072, and 0412305, respectively, for issues number 1 to 11.

Contributions. Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in \LaTeX . The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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