Genomes and Games

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Background

- Ciliates are one-celled organisms with encrypted copies of their genomes.
- Ciliates decrypt these copies using context-directed reversals(CDR) and context-directed swaps(CDS).
- We use game theory to study the CDR and CDS sorting process on permutations.

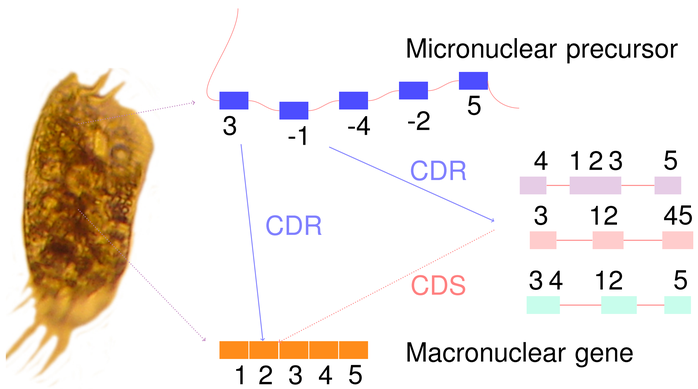

Figure 1 : The ciliate species Stylonychia Lemnae has a complex micronuclear decryption process.

Objectives

- Understand the mathematical aspects of DNA sorting
- Determine which player has a winning strategy in CDS and CDR games

CDR and CDS

Pointers (i,i+1) are used to make moves.

Let $\alpha = [_{(2,3)}3, -1, -4, _{(2,3)} - 2, 5]$
2 and 3 have opposite sign in α.
So CDR (2,3) can be done to α, resulting in:
$[4, 1, -3, -2, 5]$

Let $\beta = [_{(2,3)}3, _{(5,6)}6, 2_{(2,3)}, 4, 5_{(5,6)}, 1]$
The pointers (2,3) and (5,6) overlap in β, and **2, 3, 5**, and **6** all have the same sign.
So CDS (2,3),(5,6) can be done to β, resulting in:
$[4, 5, 6, 2, 3, 1]$

CDR and CDS Games

- Ciliate sorting operations applied to permutations produce fixed points that need not be desired sorting outcome - Figure 1.
- Two-player games model this dynamic of ciliate sorting operations.
  - Desired sorting outcomes are assigned to player ONE, the rest to TWO.
  - ONE and TWO alternately apply a ciliate sorting operation.
  - ONE wins a play if the fixed point reached is as desired. Else, TWO wins.

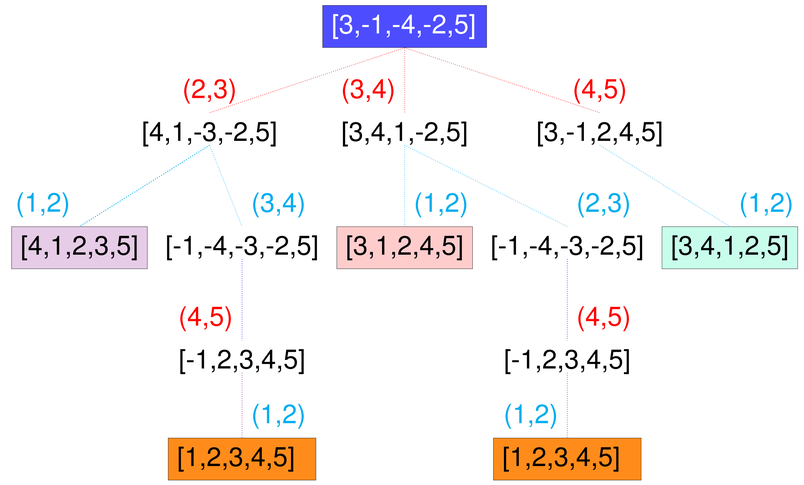

Figure 2 : A game tree for CDR moves. Game trees catalogue all possible histories of a game and aid in strategic considerations.

Move and Overlap Graphs

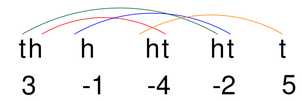

Figure 3 : Overlap graph contruction [3,-1,-4,-2,5]. Pointer diagrams aid in determining legal moves for the players.

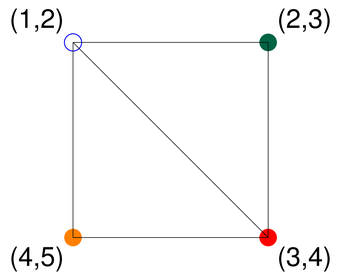

Figure 4 : Overlap Graph of [3,-1,-4,-2,5]. Overlap graphs visually catalogue legal position-moves.

Results

CDS Rescue Theorem
Fixed points of CDR-sortable permutations are CDS sortable.

CDR Parity Theorem
All branch lengths of a CDR game tree have the same parity.

Path Length Equivalence Theorem
All paths from α to β are of the same length.

CDS Game Optimality Theorem
For $\alpha \in \mathbf{S_n}$ and $\boldsymbol{P}$, the set of cds-fixed points of α, consider $\boldsymbol{A} \subset \boldsymbol{P}$. In the game **CDS($\alpha$, $\boldsymbol{A}$)**, TWO has a winning strategy if $|\boldsymbol{A}| < \frac{|\boldsymbol{P}|}{4} - \mathbf{2}$. However, if $\boldsymbol{n} - \mathbf{1} \equiv \mathbf{3} \mod \mathbf{4}$, there is an α and $\boldsymbol{A}$ such that $|\boldsymbol{A}| = \lceil \frac{|\boldsymbol{P}|}{4} \rceil$ and ONE has a winning strategy.

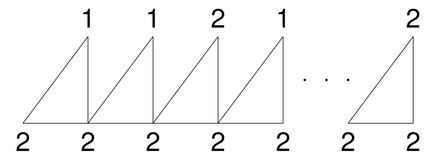

Figure 5 : Move graph of size $\boldsymbol{4k - 1}$ where ONE has a winning strategy

Future Work

- Determine the complexity of CDS and CDR games
- Determine which classes of graphs are the move graphs
- Consider games on graphs in more generality

References and Acknowledgements

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