Geometry, Topology, and Complexity of Virtual Knots

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In collaboration with:

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Historical Background

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- chemistry
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- computing

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Two knots are equivalent if one can be transformed into the other via a deformation of R^3. i.e if we can make one knot into the other without cutting or passing the knot through itself.

Knot Diagram

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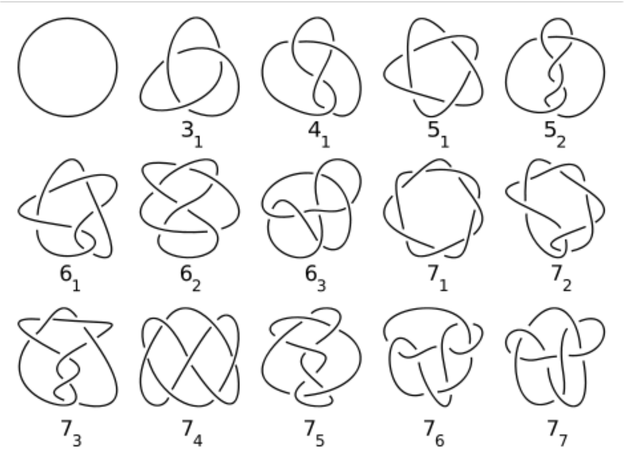
- We can think of it as the projection of a knot onto the plane.
- It is a 4-regular planar graph with over- and under- crossing information at each vertex.
- A given knot have many different knot diagrams. Equivalently, different knot diagrams may or may not correspond to different knots.

Examples



Knot Diagrams cont.

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Question: Given different knot diagrams, how do we know whether they represent different knots or not?

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(a) (b) (c)

Reidermeiser Moves

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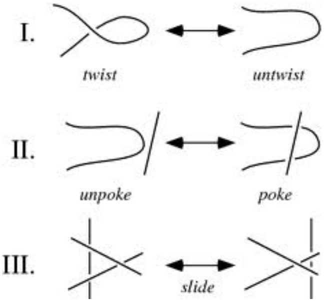
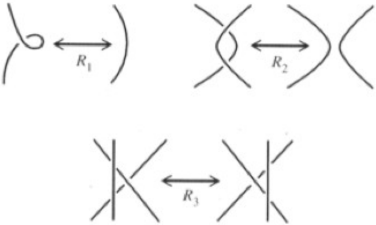
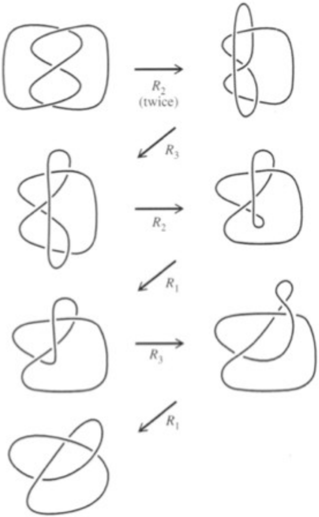
The three types of Reidermiser moves are:



Example





Knot Invariant- Tricolorability

Definition

A knot diagram is tricolorable if each of the strands in the diagram can be colored one of three different colors, so that at each crossing, either three different colors come together or all the same color come together. We also require that at least two colors are used.

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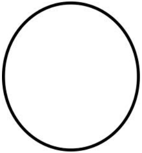
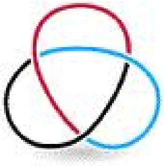
It can be shown that the property of tricolorability is an invariant under Reidermiser moves. Therefore, either every diagram of a knot is tricolorable or no diagram of that knot is tricolorable.

Example



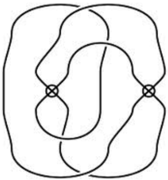
Not tricolorable



Tricolorable

Virtual Knot Diagram

The study of virtual knot is a generalization of knot theory. Unlike knot diagrams, virtual knot diagrams are 4-regular graphs, but not necessarily planar, with over- and under- crossing information at the vertices.



Acknowledgements

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