Geometry, Topology, and Complexity of Virtual Knots

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Diagramatic Reducibility

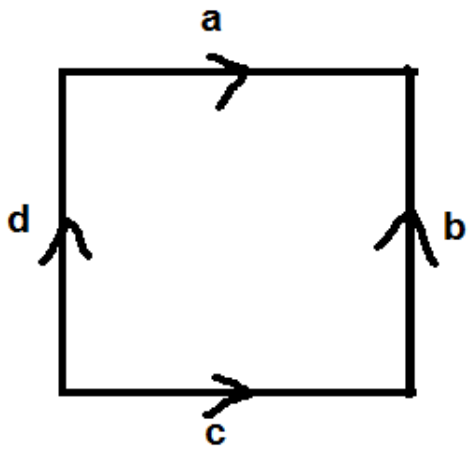
Definition

A Spherical Diagram is **Reducible** *if it contains a sub-tiling.*

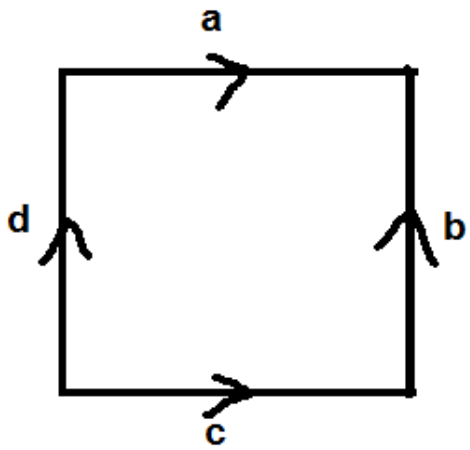
Definition

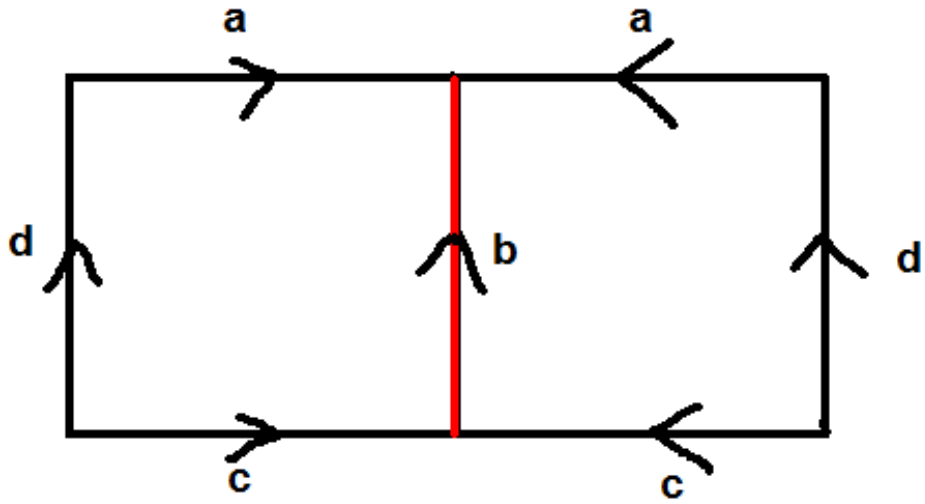
A 2 complex is **Diagramatically Reducible (DR)** *if it only admits reducible spherical diagrams.*

A Simple Example



A Simple Example





Combinatorial Curvature of a Face

Given an k sided figure we can arbitrarily apply to each of the k corners an "angle measure".

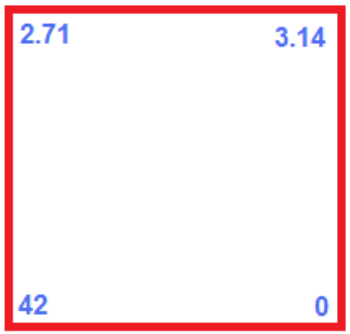


Figure : A Valid angle assignment

Let $\{\beta_i\}$ be the set of angles of the figure. Then its combinatorial curvature is $\Sigma\beta_i - (k-2)\pi$.

Combinatorial Curvature of a vertex

Given a tiling of a figure we can also define the curvature around a vertex by summing the angles of the faces around that vertex then subtracting 2π.

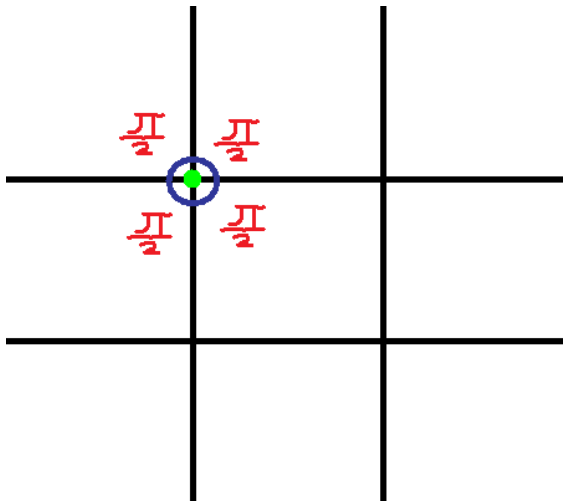


Figure : The green vertex has curvature 0

The Weight Test

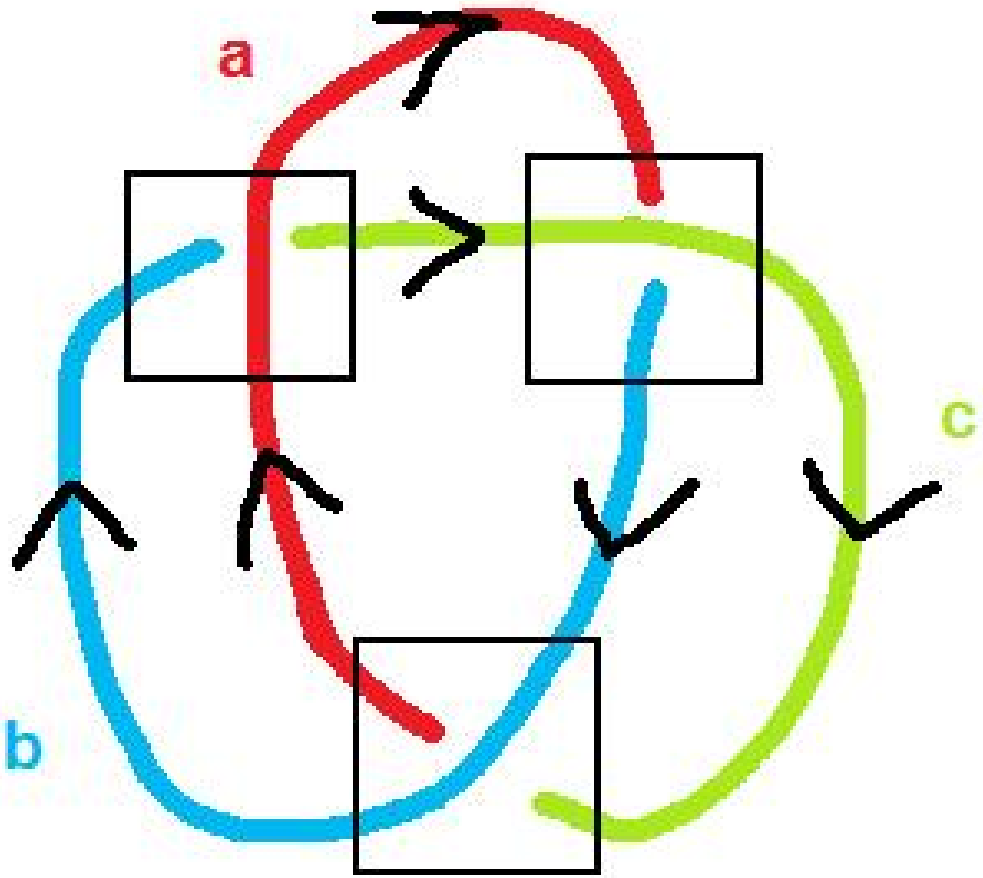
Gauss Bonet Formula

$\Sigma VertexCurvature + \Sigma FaceCurvature = 2\pi\chi(x)$

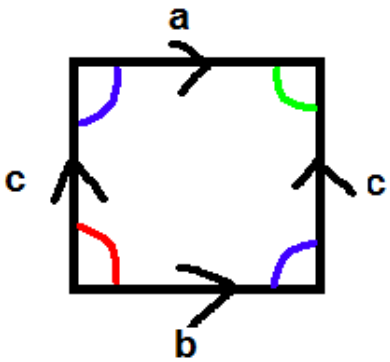
Corollary (Weight Test)

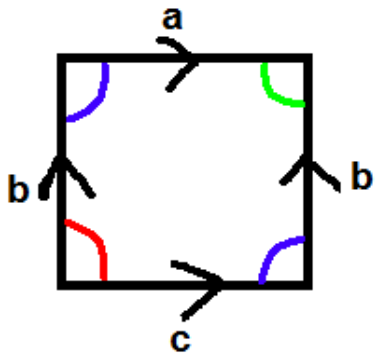
Suppose that a 2-Complex K with two cells $\{d_i\}$ has been assigned a combinatorial angle measurement. Then if $\forall i$ d_i has curvature ≤ 0, and all possible vertices of a tiling using the 2 cells of K have curvature ≤ 0. Then K is Diagramatically Reducible.

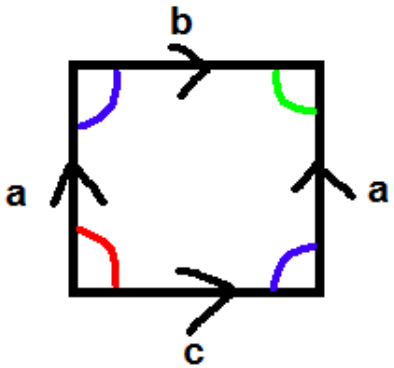
The Trefoil is DR



The Trefoil is DR







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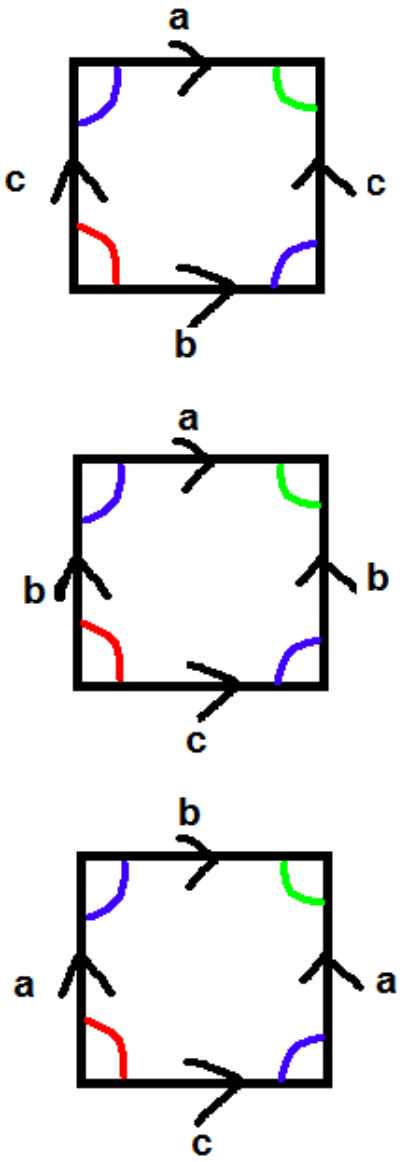
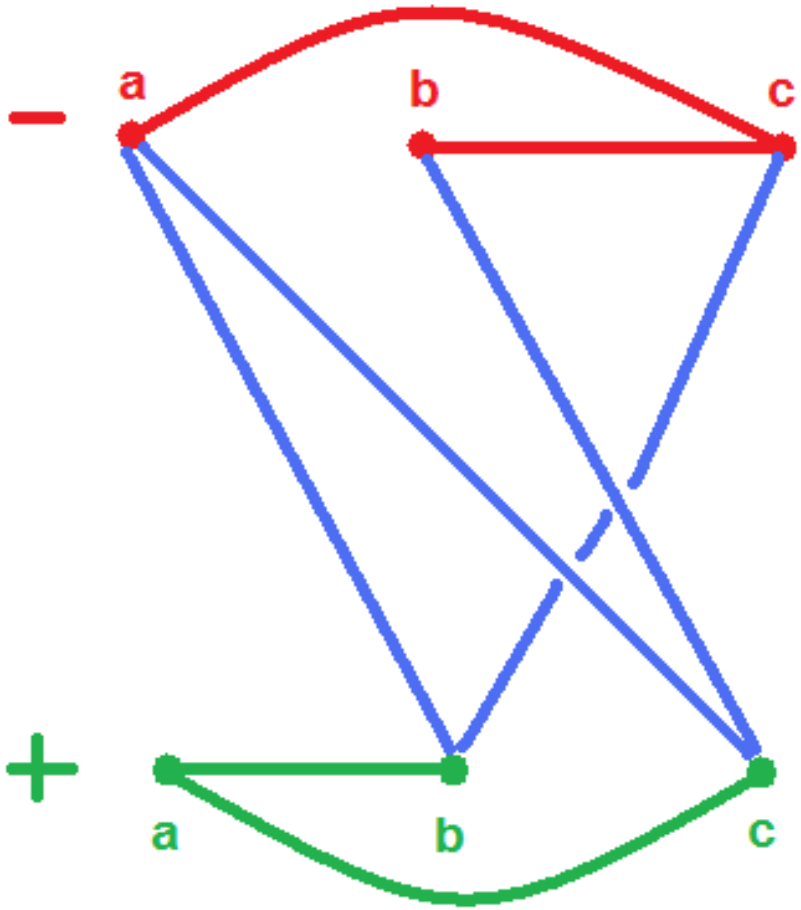
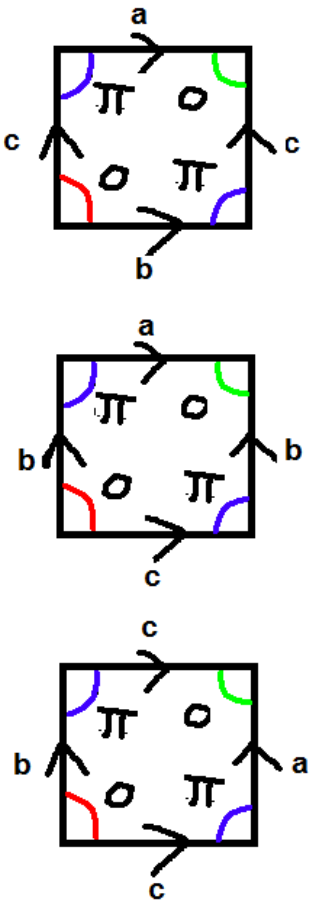
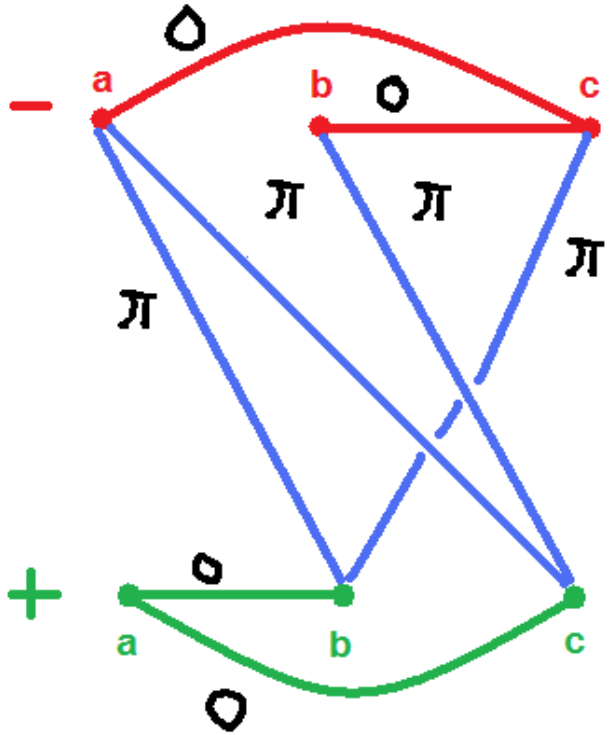




Angle Assignments





Reduced Diagram

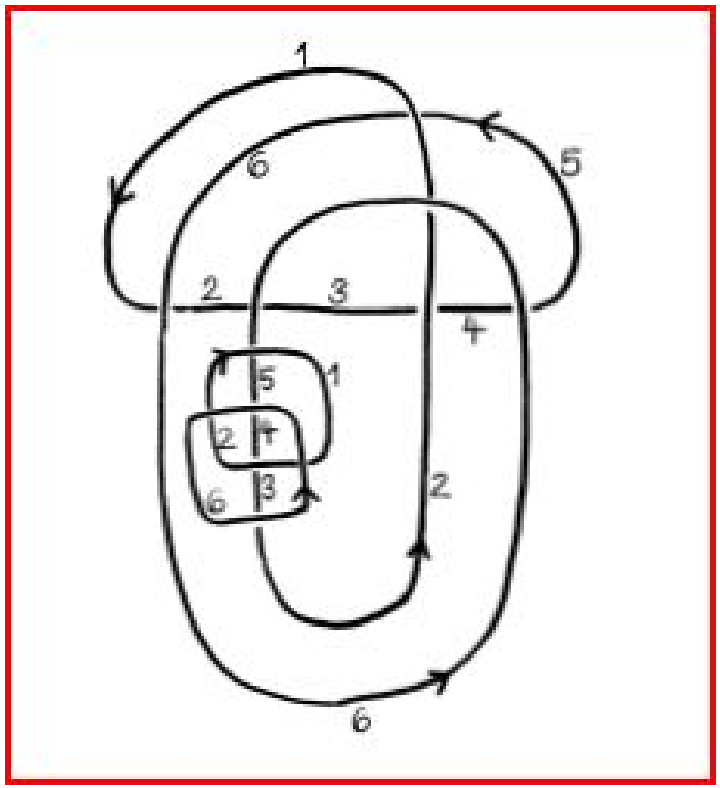


Figure : The smallest known reduced diagram.

Searching for Reduced Diagrams

Reduced spherical diagrams are rare.

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Reduced spherical diagrams are rare.
Is this complex diagramatically reducible? Undecidable

Searching for Reduced Diagrams

Reduced spherical diagrams are rare.
Is this complex diagramatically reducible? Undecidable
Does there exist a general construction principle?

Acknowledgements

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 [Jens Harlander, 2012]
REU Lectures on Combinatorial Topology, 2012.

 [Stephan Rosebrock, 2011]
Some Spherical Diagrams over Labeled Oriented Trees and Graphs, 2011.