

Geometry, Topology, and Complexity of Virtual Knots

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The material presented today is currently being studied by our research team:

Gabriel Islambouli (University of Virginia)

Rachael Keller (Louisiana State University)

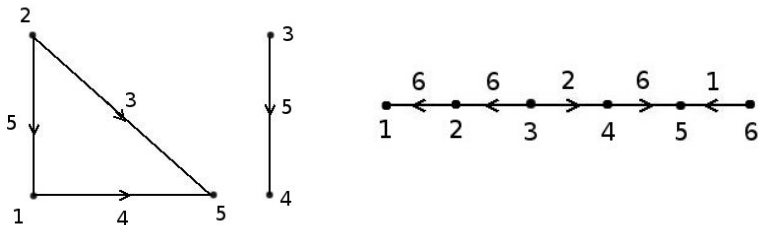
Mingjia Yang (Albion College)

Mentor: Dr. Jens Harlander (Boise State University)

LOGs and LOIs

A labelled oriented graph (LOG) is a directed graph on vertices $\{1, \dots, n\}$ where edges are labelled by vertex labels.

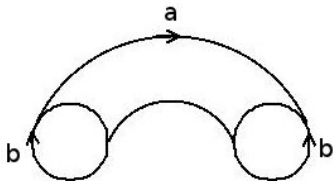
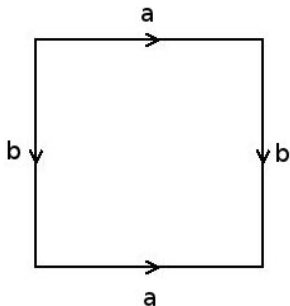
A labelled oriented interval (LOI) is a LOG whose underlying graph is an interval.



In this talk we will deal only with LOIs.

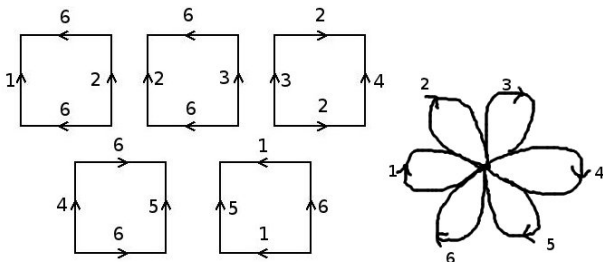
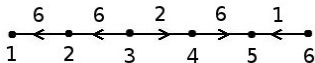
2-cells

A 2-cell is a polygon whose edges are labelled. The inside of a 2-cell can be thought of as an open disk.



2-cells

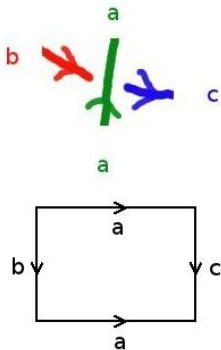
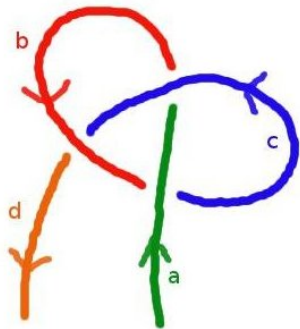
Each edge of a LOI encodes a 2-cell:



A 2-complex is an object that can be built from gluing 2-cells to 1-cells.

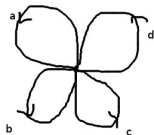
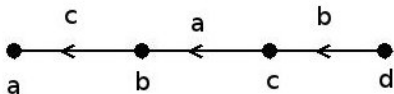
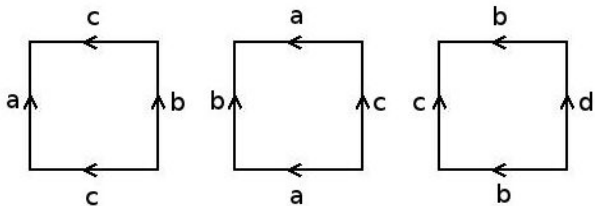
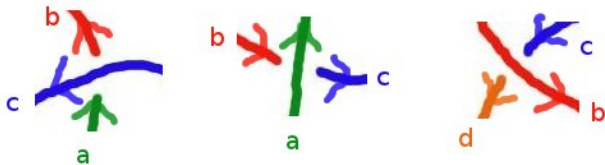
2-complexes

Every knot (classical and virtual) has a 2-complex associated with it.



2-complexes

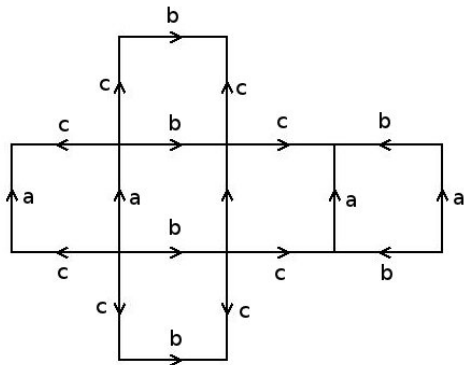
After doing this for all the crossings:



This is called the Wirtinger complex of a knot (denoted by $W(k)$)

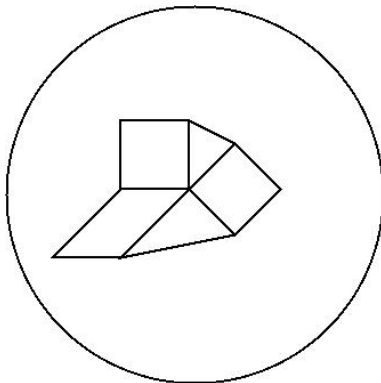
Spherical Diagrams

Specifically, we are concerned with constructing spherical diagrams. These are surfaces constructed by gluing together 2-cells, which can then be deformed into a sphere.



Spherical Diagrams

We can also think of spherical diagrams as a tessellation of the surface of a sphere with polygons.

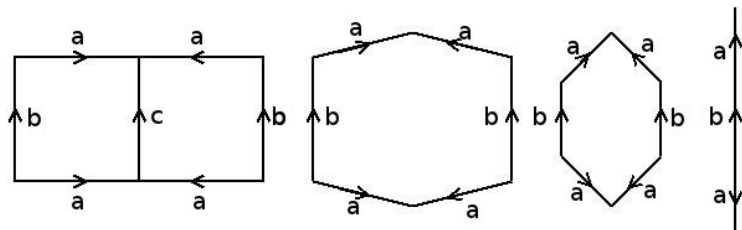


Spherical diagrams that we get from knots will only contain squares.

Reducing Spherical Diagrams

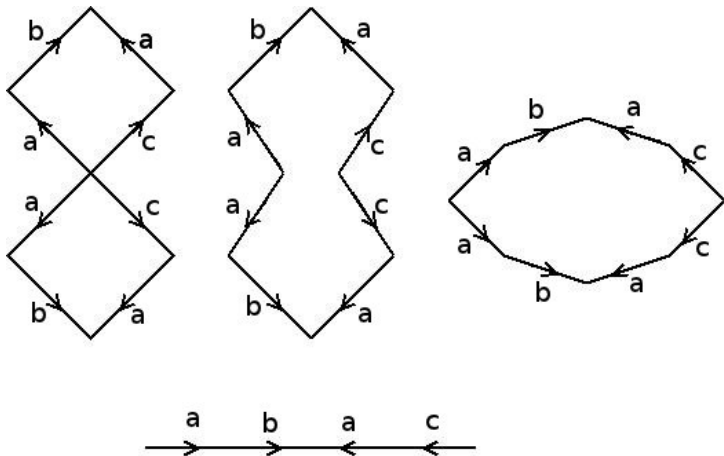
Given a spherical diagram, it is sometimes possible to reduce the number of tiles used by reorganizing them. There are two ways to do this.

Edge fold:



Reducing Spherical Diagrams

Vertex fold:



Algebraic Topology, Allen Hatcher

Some Spherical Diagrams over Labelled Oriented Trees and Graphs,
Stephan Rosebrock

Acknowledgements

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