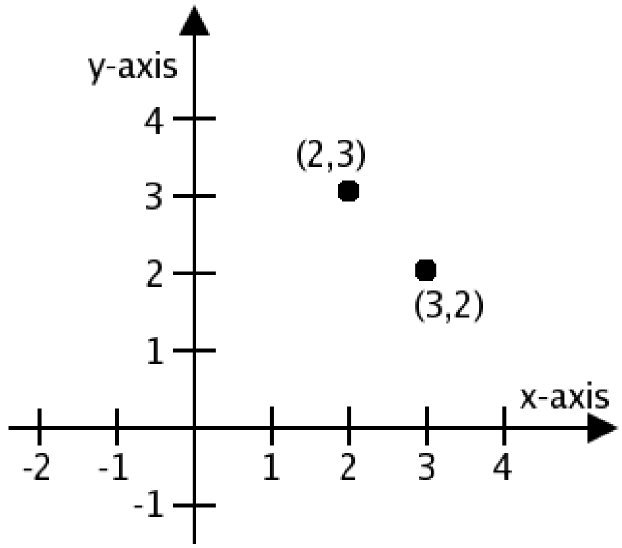
Task #1: Draw Some Points



One way to describe points on a plane is to create *x* and *y* axes, and label points using coordinates (as shown in the picture).

a) Draw a small figure near the origin on the provided piece of graph paper.

b) Then make a 'connect the dots' frame for the figure by marking about 10 dots.

c) List the coordinates of your dots below:

Task #2: Move the Points

A **linear transformation** is a rule that mixes the coordinates of points in order to make new points. These kinds of transformations have rules that all look like:

$$(x, y) \longmapsto (\ a \cdot x + b \cdot y\ ,\ \ c \cdot x + d \cdot y)$$

Example: The linear transformation $(x, y) \longmapsto (\ 2 \cdot x + -3 \cdot y\ ,\ \ 1 \cdot x + 1 \cdot y)$ will turn the coordinates $(1, 2)$ into the point $(-4, 3)$.

a) Write down the coordinates of your dot figure after applying the linear transformation in the example. Then 'connect the new dots' on your graph paper to see what the transformation did to your drawing.

Task #3: Move a Square

We often abbreviate a linear transformation by just keeping track of the constants. So, the rule $(x, y) \longmapsto (\ a \cdot x + b \cdot y\ ,\ \ c \cdot x + d \cdot y)$ can be written as:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

a) Draw the square with the coordinates $(1,1)$, $(1, -1)$, $(-1,1)$, and $(-1, -1)$ on some graph paper.

b) Find out what each of the following transformations does to the square by computing the new locations of the corners and 'connecting the new dots'.

1. Scale:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

2. Shear:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

3. Shear/Scale:

$$\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$$

4. Rotate/Scale:

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

5. Double Shear/Scale:

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

6. Arbitrary:

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

Task #4: Calculate Areas

Calculate the areas of the shapes made by the 6 transformations in the previous task. What do you notice?

Task #5: Move a Square, then Move it Again

a) What will be the final coordinates for each of the four points of the square in Task #3 after you apply transformation 2, and **then** apply transformation 6 to the result?

b) What will be the final coordinates for each of the four points of the square in Task #3 after you apply transformation 3, and **then** apply transformation 6 to the result?

c) What will be the final coordinates for each of the four points of the square in Task #3 after you apply a generic transformation like the one below, and **then** apply transformation 6 to the result?

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

d) Try to come up with a formula that will tell you the final coordinates of a point after you apply one transformation, and then apply another transformation to the result.

Extension Tasks

a) What is the final result of applying the following transformation ten times in sequence to a point (x, y)?

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

b) What is the final result of applying the following transformation ten times in sequence to a point (x, y)?

$$\begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

c) Can you invent two transformations A and B so that A-followed-by-B = B-followed-by-A? Prove it.

d) Can you invent two transformations A and B so that A-followed-by-B does not give the same result as B-followed-by-A? Prove it.

e) Find the transformation that always 'undoes' the following?

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

f) Find the transformation that always 'undoes' the following?

$$\begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

g) What effect do transformations have on the *area* of figures?